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A CRITICAL REVIEW OF THE WORLD LITERATURE IN APPLIED MECHANICS AND RELATED ENGINEERING SCIENCE

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APPLIED MECHANICS REVIEWS

VOL. 5, NO. 11 MARTIN GOLAND *Editor* NOVEMBER 1952

A SURVEY OF "STATISTICAL EFFECTS" IN THE FIELD OF MATERIAL FAILURE

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THE classical theory of material failure presumes the existence of a generalized stress X, i.e., a function of principal stresses or strains, having the property that failure occurs as soon as it reaches a certain value, the ultimate strength, in any point of the stressed volume. The hypothesis that failure can be predicted from stress conditions in one single point is inconsistent with many observations. Its validity has also been doubted for many years (1).[1]

Any simple test, repeated sufficiently often with nominally identical specimens, shows a scatter of the ultimate strength, caused by many unknown factors. This scatter, though often neglected, constitutes an intrinsic property of all real materials. The strength is thus a statistical quantity which has to be defined, not by a single figure, but by a statistical distribution function (2). Many "effects" which are incompatible with the classical theory may be explained by this concept. They will be called "statistical effects."

A systematic classification may be obtained in the following manner: If P is the probability that $X \leqslant x$ in a small volume dV at the moment of failure, then it is possible to put, without loss of generality

$$P = 1 - e^{-f(x)dV}$$

from which it follows (3) that the probability of $X \leqslant x$ in a volume $V = \int dV$ is

$$P = 1 - e^{-\int f(x)dV} \qquad [1]$$

where, in general, $f(x)$ is a function of the coordinates of dV.

The traditional way of computing the ultimate strength from a number of observations has always been to identify it with the arithmetic mean x_m of the individual observations. But, since

$$x_m = \int x dP \qquad [2]$$

there will be three principal ways of influencing its value, namely, either by changing V, i.e., the geometrical conditions (size and shape), or by changing X, i.e., the stress system (distribution and type), or by changing the function f or its parameters, i.e., the material.

A change in material does not originate a statistical effect as this influence on the strength is part of the classical concept and has, in fact, very often been proposed as an explanation of effects which were in reality of a statistical nature.

Thus, statistical effects may be arranged in four classes, corresponding to influences of size, shape, stress distribution, and stress type (combined stresses). It seems practical to add a fifth class for notch and stress-concentration effects, since these effects are composed of all the other four, which are almost impossible to separate from each other.

It may be noted that the origin of statistical effects is a consequence of the simple but quite arbitrary practice of defining the ultimate strength as the arithmetic mean of observed values. A value x_u, defined by $f(x_u) = 0$, is completely unaffected by changes in size, shape, and stress distribution and should for this reason (and others) be a better measure of the ultimate strength than the classical one. (A strength corresponding to a very low probability, for instance, $P = 1\%$ or 0.1%, is practically unaffected by the above-mentioned factors and might be used as a substitute for x_u, whether its value is unknown or equal to zero.)

All the preceding considerations are directly applicable to fatigue problems by exchanging, for the ultimate strength, the fatigue strength at a specified life N, i.e., the two limits X_0 and X between which the generalized stress must pulsate in order to give failure after N stress cycles. The ultimate strength may thus be regarded as a special case ($X_0 = 0$, $N = 1$).

SIZE EFFECTS

This effect has been proved in a great many static tests with specimens of *brittle materials* such as rock salt (4), plaster of Paris (5, 6), glass (7, 8, 10), crystalline minerals (8, 9), steel at low temperatures (11), cast iron (12, 13, 14), and porcelain (15).

It may be mentioned as most remarkable that thin glass fibers have a tensile strength much higher than steel (7), that the breaking load on a ball pressed against a plate is proportional to the diameter of the ball (8, 9, 10) which is in sharp contrast with all classical rupture criteria (3, p. 20), and that cast iron shows a surface effect which overlaps the volume effect (14). The volume and the surface effects may be separated by statistical mean as demonstrated on porcelain rods (15, pp. 47–49).

The earliest observations of size effects in *ductile materials* were made on wires of many different metals by varying the diameter

[1] Numbers in parentheses refer to the References at end of the paper.

(16, 17). The influence of the length of the wire (18), of the diameter of cylindrical tensile test specimens (19), and of the width of ship plates (20) has also been demonstrated.

In some cases, the change of dimensions may affect the properties of the material as well; e.g., as in cold drawing of wires or casting of iron specimens. The size (volume or surface) effect and the material effect may be separated by plotting loglog $1/(1 - P)$ versus any function of x as previously demonstrated (15, p. 13).

The law of similitude (15, p. 12) has been verified by tensile tests on steel specimens of complicated shape (3, pp. 31, 42), and by bursting spherical rotors by centrifugal forces (21).

Size effects *in fatigue* have been studied in many investigations (22–31). It is a well-known fact that the surface condition plays an important role in fatigue. The existence of surface effects such as found in cast iron (14) is very plausible. A closer examination of this problem is of fundamental importance and might be conducted in the same way as already mentioned in connection with porcelain rods (15, pp. 47–49).

As the scatter of fatigue life, in general, is very large, one must be warned against basing conclusions about size effects on too small a number of observations (32). Evidently, the relation between size effect and scatter holds good only in so far as the scatter is due to the material, and not to the measuring technique or other irrelevant factors.

SHAPE EFFECTS

A necessary condition for a shape effect is a nonuniform stress system. As the stress distribution is not very well known at the moment of failure, in ductile materials, and the law of similitude cannot, for obvious reasons, be applied in this case, shape effects have been observed only in static tests with *brittle materials* such as cast iron (33), plaster of Paris (5, 6), and sugar (15, p. 46), and *in fatigue* (22, 24, 27, 31, 34, 35).

STRESS-DISTRIBUTION EFFECTS

It is an old experience that materials are stronger in bending than in tension. This effect has been observed in many *brittle materials* such as glass (36), stearic acid (37), plaster of Paris (5, 6), ceramics (38), steel at low temperatures (11), cast iron (12, 13, 14, 39), and bricks (39).

The interpretation of results from static tests with *ductile materials* is difficult due to plastic flow, and *in fatigue* due to surface effects. So far no attempts have been recorded.

STRESS-TYPE EFFECTS

The experience that combined stresses have an effect on the material different from that of pure tensile stress is a very old one. As a matter of fact, it has for a long time been a dominating problem to find a general stress criterion of failure.

From a statistical viewpoint the principal problem may be formulated as: To find the distribution functions of any given generalized stress X for the most important materials. This is an experimental task, as yet not even started.

Another problem of a more theoretical nature will be to deduce the effect of bi- and triaxiality on the distribution functions of one-dimensional stresses. If the principal stresses act independently of each other, and it seems that such materials may exist, the problem may be quite solvable along lines previously sketched (2, pp. 14–16). In other cases, the solution is very intricate and will certainly require a close examination of the physical behavior of the material in question. Some recent publications, even if not initiated by statistical motives, regarding *brittle materials* (10, 36, 37, 40), *ductile materials* (41–45), and *fatigue* (46–49) may be mentioned.

It deserves attention that due consideration has to be taken of the size effect when investigating the influence of combined stresses. A given specimen, for instance, has a much larger stressed volume in tension than in torsion.

NOTCH EFFECTS

There are many evidences that notches and similar stress concentrations are less dangerous than is predicted by the classical theory. This is quite understandable, as the volume subjected to high stresses, in general, is very small. A convincing example has been given by testing *plaster* specimens with holes of decreasing diameters (50).

The effect in *ductile materials* is to a great extent a question of plastic flow. The statistical part of it may be studied by applying the law of similitude (51). Notch-bend tests on geometrically similar specimens (52), and bursting tests of rotating disks with a small central hole (53) have been carried out.

Various types of notches have been tested *in fatigue* (54–63). In this connection, an interesting theory has been proposed (61).

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Communications

Correction to Revs. 2248 and 2249 (August 1952).

The correct transliteration of the authors' names is Mikhlin and Tikhonov.

Theoretical and Experimental Methods

(See also Revs. 3024, 3027, 3061, 3114, 3117, 3129, 3169, 3231, 3243)

2979. Lavrent'ev, M. A., and Sabat, B. V., Methods of the theory of functions of a complex variable [Metodí teorii funktsií kompleksnogo peremennogo], Moscow-Leningrad, Gosud. Izdat. Tekh.-Teor. Lit., 1951, 606 pp.

Book is written for students of physical and technical faculties of the Russian universities. Reviewer considers it a very successful work. Avoiding complicated theoretical constructions, it presents briefly and clearly all necessary mathematical equipment and shows, in an abundant variety of examples, application of the theory in all main parts of engineering and physical sciences.

Material is divided into seven chapters. In the first, in addition to usual subjects, expansion in series and theory of Riemannian surfaces are explained. The next main part is devoted to conformal transformations. Theory is abundantly illustrated with examples. Especially recommended to the reader are sections on the principle of symmetry and on representation of polygons.

Third chapter deals with boundary-value problems. It gives a fine summary of the subject and explains by many examples (harmonic functions, problems from different parts of physics and engineering, technical use of integrals analogous to those of Cauchy, etc.) the role of complex variables in applied science. The section about variational principles in conformal representation has the same significance. It is the question of how much and where to vary a construction if some of its dimensions extend over the admissible limit. Methods of the fourth chapter give information of this kind. The following main part explains the use of complex variables in analysis. Chiefly noted are sections on the residues and methods of asymptotic estimations.

Operational calculus is treated in the sixth chapter, and the last section deals with special functions. Names of paragraphs: Function gamma; orthogonal polynomials; cylindric functions; elliptic functions.

Each chapter has a literature index. Misprints are numerous but do not disturb the reading. Reviewer gladly recommends the book to mathematicians, engineers, and physicists.

V. Vodička, Czechoslovakia

2980. Tolstov, G. P., Fourier series [Ryadi Fur'e], Moscow-Leningrad, Gosud. Izdat. Tekh.-Teor. Lit., 1951, 396 pp.

Book is designed for engineers and represents an introduction to the theory of trigonometric series. It presupposes only mathematical knowledge as covered by courses in technical high schools.

Content is divided into 11 chapters. Reader's attention is called especially to sections on convergence and summation of Fourier series as considerations on their differentiation and integration. Also, double trigonometric series and Fourier integrals are treated in a clear way. Two chapters are devoted to elements of Bessel functions and series of Bessel-Fourier type. Book ends with some applications of theoretical material to simpler problems in mathematical physics.

Presentation is clear and vivid. Engineers will gladly read and use this fine and instructive book.

V. Vodička, Czechoslovakia

2981. Hald, A., Statistical theory with engineering applications, New York, John Wiley & Sons, Inc., 1952, xii + 783 pp. $9.

This large volume with 25 substantial chapters covers, with relatively simple mathematics and avoiding theoretical abstractions, most of the statistical topics of immediate interest to engineers. Throughout, the observations are assumed to be stochastically independent. Growth curves, time series, and stochastic processes have been merely outlined. Index numbers are barely suggested, and Tippett's numbers do not appear. Intentionally, the work is very different from many designed for economists, biologists, educationalists, and mathematicians. Most of the numerous illustrative examples are drawn from various fields of the author's own engineering experience. For adequate reasons, emphasis is laid upon normal distributions rather than on nonparametric variables. Many sound suggestions, much worked-out material, and abundant references make the treatment especially valuable. The author has published tables and formulas in a separate volume. Albert A. Bennett, USA

2982. Hald, A., Statistical tables and formulas, New York, John Wiley & Sons, Inc., 1952, 97 pp. $2.50.

2983. Jowett, G. H., The accuracy of systematic sampling from conveyor belts, *Appl. Statistics* **1**, 1, 50–59, Mar. 1952.

Paper shows how a trial set of sampling observations may be used to calculate accuracy of given sampling scheme. Calculations are based on theory of time series. Author applies theory to problem of estimating ash content from random samples of coal taken along conveyor belt. Method is also applicable to statistical analysis of hardness readings at fixed intervals along steel rods, involving within-zone variation and gradual changes from zone to zone. D. E. Hardenbergh, USA

2984. Maxwell, J. C., The scientific papers of, New York, Dover Publications, 1952, vol. 1, xxix + 607 pp., vol. 2, vii + 806 pp., bound in one. $10.

Those scientists and research engineers who have never read the papers of Maxwell have in store a considerable treat; and, for those interested in the history and development of thought and ideas in the physical sciences, the papers should be required reading. His extremely lucid and often entertaining manner of writing makes pleasurable reading and is in striking contrast to the stilted style of the larger part of present-day papers.

This combined volume is a reprint of the 1890 edition which was published as a memorial to perhaps the greatest physicist of the 19th century. The introductory preface by W. D. Niven gives a short account of Maxwell's life and the sequence in which his papers first appeared. The collection includes his works on the stability and motions of Saturn's rings (the first of which was written when he was 18 and first showed his analytical power), the famous series leading up to and including his formulation of the dynamical theory of the electromagnetic field, his works on the kinetic theory of gases, work in elasticity and the important reciprocity theorems for frames and beams, his researches on color and in the field of optics, and his papers on rigid-body dynamics.

Also included are a lecture on the viscosity of gases and a delightful lecture on the telephone, which was evidently illustrated with a model. Other works include his several semipopular articles for the Encyclopedia Britannica and his reviews of treatises of his day, as well as essays on Faraday and Helmholtz which appeared in *Nature*. There are 101 contributions included in the volume. It is clear from his writings that Maxwell combined great analytical power with a talent for careful and significant experimentation as well as possessing a gift for exposition.

Not included are his treatises on "Theory of heat" (1871), "Electricity and magnetism" (1873), "Matter and motion" (1876), "The electrical researches of Henry Cavendish, etc." (1879), and "Elementary treatise on electricity" (published posthumously in 1881). Phillip Eisenberg, USA

2985. Braun, I., and Reiner, M., Note on dimensions in tensor analysis, *Bull. Res. Counc. Israel* **1**, 4, 81–82, Feb. 1952.

In tensor analysis, three different choices for the dimensions of coordinates x^i are possible: (1) all x^i are dimensionless; (2) all x^i have dimension of length; (3) each x^i has the dimension of the geometrical quantity which it represents. For each system, the dimensions of the components of the covariant and the contravariant metric tensor are determined by aid of the expressions for the linear element.

Authors show the following properties: (*a*) The statement that in rectangular Cartesian coordinates the covariant and contravariant coordinates are indistinguishable holds, if dimensions are included, only in systems (2) and (3); (b) since only a dimensionless number can be used as a measure of strain, there exist only in system (2) covariant, contravariant, and mixed measures of strain. A. I. van de Vooren, Holland

2986. Manarini, M., On a useful vectorial homography (in Italian), *R. C. Semin. Fac. Sci. Univ. Cagliari* **20**, 151–154, 1951.

Author applies the linear vector function $\overline{H}(\mathbf{a}, \mathbf{b})$, which he calls the vector dyad, defined by

$$\overline{H}(\mathbf{a}, \mathbf{b})\mathbf{u} = (\mathbf{a} \wedge \mathbf{u}) \wedge \mathbf{b}$$

to express the moment of inertia, momentum, and kinetic energy of a system of particles in terms of $\overline{H}(P - O, P - O)$. The results differ only in the notation from those expressed by the moment of inertia tensor. L. M. Milne-Thomson, England

2987. Leutert, W., and O'Brien, G. G., On the convergence of approximate solutions of the wave equation to the exact solution, *J. Math. Phys.* **30**, 4, 252–256, Jan. 1952.

Authors substitute the classical difference equation for the two variables (x,t) wave equation with an *arbitrary* mesh ratio $r = \Delta x/\Delta t$, and establish a convergent solution v of the difference equation for any positive value of r for the Cauchy problem with boundary conditions $u(0,t) = u(1,t) = 0$, $t \geqslant 0$. The initial values $v(x,0)$ on the mesh points need not coincide with the exact values $u(x,0)$. The initial values $u(x,0)$ being developed in a Fourier series, the initial values $v(x,0)$ are given by this series limited to the k term, where k is the greatest positive integer such that $k^4 < M$, when the interval $0 \leq x \leq 1$ is divided into M subintervals. There are misprints.

F. H. van den Dungen, Belgium

2988. Staff of the Computation Laboratory, Tables of the Bessel functions of the first kind of orders seventy-nine through one hundred thirty-five, Cambridge, Mass., Harvard Univ. Press, 1951, 614 pp. $8.

This is the last of a series of twelve volumes [*Annals* **3**–**14**] which tabulates values of the Bessel functions of the first kind. This is the first review to appear in APPLIED MECHANICS REVIEWS dealing with this series, and, in view of the remarkable contribution to science afforded by these volumes, reviewer takes this opportunity to review the complete set. The range covered by the volumes is as follows: $n = 0(1)3$, $x = 0(.001)25(.01)100$, $18D$; $n = 4(1)15$, $x = 0(.001)25(.01)100$, $10D$; $n = 16(1)135$, $x = 0(.01)100$, $10D$. For $n > 135$, $J_n(x) < 10^{-10}$ for $0 \leq x \leq 100$. A detailed description for interpolation in the entire range is given in the *Annals* **3** and **5**.

The calculations were performed on the Automatic Sequence Controlled Calculator. Its electromagnetic typewriters printed not only the entries given in the tables but also 10 differences in every case which were used for check purposes. The printed values were used for offset reproduction. The typography of the volumes is excellent. Of the multitude of entries, only one error has been found [see *Math. Tables and other Aids to Computation* **3**, p. 41]. The slip was due to reproduction difficulties, not to computation or automatic checking. Most of the entries in this sequence are new. The Harvard tables are an indispensable aid to the applied mechanics worker. Y. Luke, USA

2989. **Horner, J. G., Dictionary of mechanical engineering terms. Part I. Modern terms. Part II. General and traditional terms,** London, The Technical Press, Ltd., 1952, iv + 417 pp. 22s 6d.

2990. **Krílov, A. N., On some differential equations of mathematical physics with applications to technical problems [O nekotoríkh differentsialníkh uravneniyakh matematicheskoĭ fiziki imeyushchikh prilozhenie v tekhnicheskikh voprosakh], 5th** ed., Moscow-Leningrad, Gosud. Izdat. Tekh.-Teor. Lit., 1950, 368 pp., 60 figs., 20 numerical tables.

A promising attempt at giving, within a short space, a more or less adequate idea of the contents, style, and original features of this quite voluminous book is to start with a detailed account of a typical section. For this purpose, reviewer chooses chapter VI (pp. 206–228). Its subject can be termed the Fourier problem reversed: For a given Fourier series, to find an elementary essentially closed expression for its sum. The solution of this problem, due to author, represents an important step in improving the practical procedure of integration of some differential equations in cases of slow convergence of the results. Here is how Krílov develops his simple and skillful device. He branches off from Poincaré's application of a convergence theorem of Abel [Poincaré, Henri, "Théorie analytique de la propagation de la chaleur," Paris, 1895, pp. 71–82]. In the way just indicated, Poincaré arrived at conclusions, concerning uniformity of convergence of a Fourier series, based on power series developments of its coefficients in terms of the reciprocal of the order number n. If the coefficients are proportional to the first power of that reciprocal, the function in question has discontinuity points with (finite) jumps, and vice versa; if the coefficients are proportional to the square or to higher powers of that reciprocal, there are no discontinuities and the Fourier series is uniformly convergent in the entire interval considered. So far, Krílov follows Poincaré's book; occasionally, however, with more details and in broader presentation, thus removing some little difficulties. Krílov introduces his own ideas starting from the familiar integral formulas, which express the coefficients of the Fourier series, the integrals being, of course, interrupted at discontinuity points of the function. By repeated partial integration of these formulas, he arrives at representation of the Fourier coefficients in form of power series in terms of the reciprocal of the order number n; the coefficients of such a series are linear functions of the discontinuity jumps of the Fourier series and of those of the latter's consecutive derivatives. Another set of equations for the jumps follows from Dirichlet's theorem on the value of the function at a jump. From the system of equations thus indicated, Krílov obtains by elementary integration the polynomials, which represent the Fourier series in the separate partial intervals plus the remaining rapidly and uniformly convergent Fourier series of sufficiently high order in the reciprocal of n, a series which can be regarded as a correction to be superposed on the elementary function just sketched and having the same jumps as the original Fourier series. Another characteristic example is Krílov's treatment of the secular equation. The trouble encountered in this problem originates from the fact that the unknown appears in the diagonal of a determinant, which leads to practically almost insurmountable calculational difficulties. Among all attempts associated with the names of Lagrange, Laplace, Leverrier, and Jacobi for overcoming those difficulties, Krílov's method offers the simplest solution. By a calculation analogous to the one which reduces the generating system of differential equations of the problem to one differential equation of higher order, Krílov transforms the original determinant in one, where the unknown occurs only in one column (see chapter I, particularly pp. 55–73).

These examples imply the characteristic features of Krílov's book. Starting from detailed accounts of fundamental contributions of the grand masters of mathematical science of the 19th century to the theory of integration of ordinary and partial differential equations, he proceeds to a thoroughly profound discussion of the consequences of classical procedures up to the ultimate numerical results needed in engineering applications.

This book had its origin in lectures given by Krílov (1863–1946) to the students of the Academy of Naval Engineers in St. Petersburg (Leningrad), a fact which determines the leading tendencies of the volume. He was the man entrusted with the preparation of the section on the "Theory of ships" in the "Enzyklopaedie der Mathematischen Wissenschaften," **IV**, 3, pp. 519–564. This explains the abundance of problems and their valuable numerical tables, which are of great practical interest to naval engineers; e.g., the longitudinal and radial vibrations of ship cannons, ship vibrations, etc. Unfortunately, there are many misprints in the book, so that the reader must carefully check the formulas before using them. Except for this shortcoming, the book can be highly recommended to all computing mechanical engineers. I. Malkin, USA

2991. **Mukhin, I. S., Application of the Markov-Hermite interpolation polynomials for numerical integration of ordinary differential equations** (in Russian), *Prikl. Mat. Mekh.* **16**, 2, 231–238, Mar./Apr. 1952.

Author employs the Markov-Hermite polynomial interpolation formula to derive several pairs of formulas useful in the numerical integration of differential equations, especially if solved by methods proposed by the reviewer. Of each pair, the first is an open type, or extrapolatory, integration formula; the second a closed type, or interpolatory, integration formula. The paper contains pairs with first derivatives only, pairs with higher-order derivatives (one being a special case of Obrechkoff's formula), and pairs suited to second-order equations lacking first derivatives. Most of the formulas are well known. Rigorous error terms are given for each.

Courtesy of Mathematical Reviews W. E. Milne, USA

2992. **Liebmann, G., Solution of partial differential equations with a resistance network analogue,** *Brit. J. appl. Phys.* **1**, 4, 92–103, Apr. 1950.

On resistance network of axially symmetrical type, constructed and described by the author, accuracies of 1 part in 1000 to 1 part in 10,000 were obtained. The network is provided with 60 meshes in the z-direction and 20 meshes in the r-direction, and is surrounded with a termination strip. Fifty current feeding points for varying boundary conditions are provided, and boundaries need not coincide with mesh points as corrections can be made by local modification of the network. Author also explains how measurements within a mesh can be taken and how a dielectric in an electric field can be similated.

The advantages of analog type of computer, such as described

in this paper, are simplicity of construction, ease of problem preparation, and lower cost. The advantages of a high-speed electronic digital computer are its greater versatility and higher accuracy. Whether an analog or a digital computer is to be obtained depends on the diversity of problems to be solved and the amount of money to be invested in the equipment.

A. W. Jacobson, USA

2993. Stein, M. L., Gradient methods in the solution of systems of linear equations, *J. Res. nat. Bur. Stands.* **48,** 6, 407–413, June 1952.

This is an experimental comparison of some iterative methods for the numerical solution of algebraic linear systems: A class of methods to be described in a forthcoming paper by Hestenes and Stein and based upon the method of steepest descent; another method converts first to an eigenvalue problem. In each instance, a measure of nearness to the solution is set up and examined and compared, one with the others, as the iteration proceeds.

A. S. Householder, USA

2994. Brooker, R. A., The solution of algebraic equations on the EDSAC, *Proc. Camb. phil. Soc.* **48,** part 2, 255–270, Apr. 1952.

Paper reports on the use of the EDSAC (a high-speed automatic calculator at the University of Manchester) for the solution of algebraic equations. Three techniques were tried: Bernoulli's method, the root-squaring method, and the Newton-Raphson method. The author concludes that the Newton-Raphson method is the most effective.

H. Polachek, USA

2995. Erugin, N. P., Theorems on instability (in Russian), *Prikl. Mat. Mekh.* **16,** 3, 355–361, May/June 1952.

Consider the system $dx_k/dt = f_k(x_1, \ldots, x_n;\ t)$; $(k = 1, 2, \ldots, n)$, where the f_k's are continuous functions for $t \geq T$, $|x_k| < A$, and $f(0, \ldots, 0;\ t) = 0$. Author restates in a new form a certain number of theorems on instability of the unperturbed motion $x_1 = \ldots = x_n = 0$, given previously by Lyapunov [AMR **5**, Rev. 4070] and Chetayev ["Stability of motion" (Russian), Moscow, 1946] by means of functions $V(x_1, \ldots, x_n;\ t)$ with nondefinite derivative dV/dt.

Eugene Leimanis, Canada

2996. Lur'e, A. I., Operational calculus and its applications to problems of mechanics [Operatsionnoe ischislenie i ego prilozheniya k zadacham mekhaniki], 2nd ed., Moscow-Leningrad, Gosud. Izdat. Tekh.-Teor. Lit., 1951, 432 pp.

This is one of a series of books on mathematics and physics for the use of engineers, and accordingly it is essentially a textbook on the applications of operation calculus to engineering problems, especially to problems of mechanical engineering. The mathematical part is slight, but there are numerous applications, treated rather fully and illustrated by examples sometimes carried through to numerical details. For all the applications encountered in this book it is sufficient to consider operational images of sectionally continuous functions which are dominated by an exponential function $M \exp(s_0 t)$. For such a function $f(t)$ the operational image is

$$F(p) = p\,{}_0\!\int^{\infty} e^{-pt} f(t)dt$$

where $p = s + i\sigma$ is a complex parameter and the integral is certainly absolutely convergent when $s > s_0$. The notation $F(p) \rightarrow f(t)$ or $f(t) \leftarrow F(p)$ is used for this relationship. A brief summary of the contents will indicate the scope of the book.

Chap. I: Definition of the operational image and the so-called rules of operational calculus, expansion theorems, evaluation of a considerable number of operational images, Heaviside's unit function and its application to the evaluation of the operational images of various types of discontinuous and jump functions. Chap. II: Applications to (ordinary) linear differential equations with constant coefficients, systems of such equations, and to difference equations. Chap. III: Applications to problems of mechanics and elasticity which lead to a single differential or difference equation. Chap. IV: Mechanical systems of a finite degree of freedom (leading to systems of ordinary differential equations). Chap. V: Systems of infinite degree of freedom (leading to partial differential equations). Chap. VI: The complex inversion formula and its applications. Chap. VII: Miscellaneous applications, including the solution of integral and integrodifferential equations by means of the operational calculus, and transversal vibrations of beams and plates.

Courtesy of Mathematical Reviews A. Erdélyi, USA

2997. Putnam, C. R., On the least eigenvalue of Hill's equation, *Quart. appl. Math.* **9,** 3, 310–314, Oct. 1951.

Problems dealing with the propagation of waves in periodic media lead to Hill's equation

$$x'' + [\lambda + f(t)]x = 0, \text{ with } f(t) \sim \sum_{n=-\infty}^{\infty} c_n e^{2\pi i n t},\ (c_{-n} = \bar{c}_n)$$

λ is a real parameter and $f(t)$ a real-valued continuous periodic function for $-\infty < t < \infty$, which may be represented by the above Fourier series, if the unit of the t-axis has been chosen properly. The solutions are to be quadratic integrable functions.

Author endeavors to find upper bounds for the least proper value μ of the above equation. For that purpose he starts from Rayleigh's minimum principle modified for the infinite range of t.

$$\mu = \lim_{T\to\infty} \{\text{g.l.b.}\ [{}_T\!\int^{\infty}(u'^2 - fu^2)\,dt/{}_T\!\int^{\infty} u^2 dt]\}$$

where u belongs to the class of "Vergleichsfunktionen" which, together with their first derivatives, are quadratic integrable continuous functions just as the solutions and satisfy $u(T) = 0$. Since $f(t)$ is periodic, the expression $\{\ldots\}$ is independent of T. Therefore, the limit sign may be removed and T may be supposed to be zero. Then author demonstrates that, because of the quadratic integrability of u, a function $y_n(t)$ may be chosen such that $y_n(t) \leq 0$; $t > t_n$ for a sufficiently large t_n, and that the above expression evaluated for y_n differs from the corresponding expression for u by less than an arbitrarily small positive number, so that u may be replaced by y_n and in the upper limit of the integrals ∞ by t_n. By a further refinement, author comes to the expression

$$\mu \leq {}_0\!\int^{Q}(v'^2 - fv^2)\,dt/{}_0\!\int^{Q} v^2 dt$$

where v is a real-valued continuous function with continuous first derivatives on $0 \leq t \leq Q$ and satisfies $v(0) = v(Q) = 0$. Q is, for convenience, an arbitrary positive integer. In order to find upper bounds of μ, author chooses for v functions of the type

$$v(t) = \sum_{n=-N}^{N} a_n e^{2\pi i n (P/Q) t};\quad a_{-n} = \bar{a}_n;\quad \sum_{n=-N}^{N} a_n = 0$$

P is an arbitrary positive integer relatively prime to Q. By inserting v and f into the above expression, he obtains the relation

$$\mu \leq [(4\pi^2 P^2/Q^2)\Sigma n^2 |a_n|^2 - \Sigma_n \bar{c}_{Pn} (\Sigma_k\, a_k\, a_{Qn-k})/\Sigma\, |a_n|^2$$

from which a lot of upper bounds of μ may be constructed. If $a_1 = 1/2i$; $a_{-1} = -1/2i$; $a_n = 0$ for $n \neq \pm 1$ is chosen, the relation yields for $Q = 1$: $\mu \leq \pi^2(2P)^2 - c_0 + Re(c_{-2P})$ and for $Q = 2$: $\mu \leq \pi^2 N^2 - c_0 + Re(c_N)$ with $P = N$.

Ulrich Rost, Germany

Mechanics (Dynamics, Statics, Kinematics)

(See also Rev. 3026)

2998. **Landau, L., and Lifschitz, E., The classical theory of fields (translated from Russian by Hamermesh, M.),** Cambridge, Mass., Addison-Wesley Press, 1951, ix + 354 pp. \$7.50.

An excellent introductory exposition of electromagnetic and gravitational field theory, largely in the framework of special relativity, this book treats an impressively broad range of phenomena in an (sometimes deceptively) easy-to-read style. Tensor calculus is introduced and applied as required in the exposition. Plausibility arguments are used in lieu of proofs as a general rule, the distinction being left to the reader. Reviewer would have preferred a little more precision in setting up the tensor notation and manipulation. Translator has (according to his preface) refrained from changes beyond correction of errors. Electromagnetic waves and radiation, light propagation and polarization, and gravitation are discussed, and illustrative problems contribute to the generally good exposition.

L. B. Hedge, USA

2999. **Moroshkin, Yu. F., Determination of configuration of mechanisms** (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **82**, 4, 533–536, Feb. 1952.

A needlessly involved discussion of the trivial fact that the equations defining the relative positions of the members of a linkage can be stated in terms of the relative directional cosines of coordinate frames connected rigidly with the members. The advantages, if any, of this unfastidious approach are not mentioned.

A. W. Wundheiler, USA

3000. **Jagger, J. G., A textbook of mechanics,** London, Blackie & Son, Ltd., 1952, xx + 826 pp. 60s.; also available in parts: **Vol. I: Statics and dynamics,** 30s. **Vol. II: Mechanics and machines,** 17s 6d. **Vol. III: Elasticity and vibrations,** 17s 6d. **Vol. IV: Hydraulics,** 17s 6d.

Author presents, in older engineering style, elementary portions of particle and rigid-body mechanics, mechanics of simple machines, strength of materials (denoted as elasticity by author), linear single-degree-of-freedom vibrations (denoted as elastic vibrations), and hydraulics. Main emphasis is on techniques of solving problems. Precise definitions are rarely encountered, and little effort is devoted to developing general principles from fundamental laws. Problems are of the standard type, the great majority being numerical. A working knowledge of differential and integral calculus is assumed.

John L. Bogdanoff, USA

3001. **Reshetov, L. N., Friction in teeth with involute profiles** (in Russian), *Trudî Sem. teor. Mash. Mekh.* **1**, 70–80, 1947.

The "pressure pole" is the intersection of the line of centers with a line inclined at the friction angle to the common normal at the point of contact. The pressure pole always moves toward the pitch point. If two pairs of teeth are in contact, the resultant of the two reactions involved is determined, and its intersection with the center line is the pressure pole. The author determines the average displacement of the center of pressure in terms of the distances along the path of contact covered during single-contact and double-contact periods, in approach and recess. The coefficient of efficiency is shown to be a function of this displacement.

A. W. Wundheiler, USA

3002. **Muchnikov, V. M., On a general method of solution of the equation of motion of a train** (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **81**, 4, 521–524, Dec. 1951.

A train, considered as a uniform elastic cable with mass (locomotive) attached, passes with constant velocity over a track whose elevation is a continuous and piecewise linear function. Longitudinal displacements and stresses are studied. Article is an interesting exercise in formal operational mathematics, but several errors and omissions may lead to confusion.

R. E. Gaskell, USA

3003. **Toupin, R. A., A variational principle of the mesh-type analysis of a mechanical system,** *J. appl. Mech.* **19**, 2, 151–152, June 1952.

See AMR **5**, Rev. 332.

3004. **Morduchow, M., and Galowin, L., On double-pulse stability criteria with damping,** *Quart. appl. Math.* **10**, 1, 17–23, Apr. 1952.

It is demonstrated that a necessary condition for the stability of a system governed by an equation of the form $x'' + f(t)x' + k(t)x = 0$ where $f(t)$ and $k(t)$ are of period 2π that ${}_0\int^{2\pi} f(t)dt \geq 0$. Hence, if $f(t)$ is a simple function, i.e., assumes successively a set of constant values $f_1, f_2, \ldots f_n$, then the condition is that the weighted mean of these constants, which is equal to the weighted mean of the roots associated with each of the separated equations with constant coefficients, is not positive. For the case of a double pulse system, i.e., $f(t) = f_1$, $k(t) = k_1$, for $0 < t < \pi$ and $f(t) = f_2$, $k(t) = k_2$ for $\pi < t < 2\pi$, a necessary and sufficient condition is derived. This is done by expressing the general solution as a sum of two modes, each characterized by the geometric progression of values separated by time intervals of 2π. The two progression ratios which are expressed in terms of the four roots of the two separated equations are required not to exceed unity. It is observed that, contrary to the constant-coefficient case, the stability of a double pulse system is affected by the imaginary parts of the associated roots. In fact, for fixed values of the real parts, values of the imaginary parts exist such that instability obtains.

Leonard Pode, USA

3005. **Skuridin, M. A., The dynamics of lower pairs and the kinetostatics of two-link members, considering friction** (in Russian), *Trudî Sem. teor. Mash. Mekh.* **2**, 55–100, 1947.

The systems considered are: (1) A rectangular slider between rectilinear guides; (2) a turning pair; (3) two rigid bars connected with each other and with the rest of the mechanism by means of three turning or sliding pairs. Play is assumed throughout.

As is well known, in dry friction problems a number of cases must be examined if there is no initial slipping at some contact points; impossibility of the initial conditions, or indeterminacy may occur. With play present, the number of cases increases because various combinations of contact points are possible. The paper determines the cases possible for the three systems and shows how to define them in terms of the existence of intersections of certain lines in certain regions. It is, unfortunately, impossible to state here any specific results because of their complexity and need for diagrams. Their general nature resembles that of the familiar problem of a ladder leaning against a rough wall and a rough floor. Cases of impossibility arise often; and the author favors their interpretation by Prandtl (occurrence of shocks).

A. W. Wundheiler, USA

3006. **Zinov'ev, V. A., Design of four-bar linkages for given positions of crank and lever** (in Russian), *Trudî Sem. teor. Mash. Mekh.* **7**, 25, 69–84, 1949.

The author introduces a very minor simplification in the direct and exact solution of the (indeterminate) problems: Find four-bar linkages given (1) two pairs of simultaneous positions of crank

and lever, (2) one position and the corresponding velocity of the lever.
A. W. Wundheiler, USA

3007. Shul'gin, M. F., Poisson's theorem for the equations of dynamics with coupling factors (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **84**, 3, 453–456, May 1952.

The method of integration based on successive application of the classical Poisson theorem is extended to the equations of motion of a holonomic conservative system with coupling factors.
Eugene Leimanis, Canada

3008. Artobolevskiĭ, I. I., Blokh, Z., and Dobrovol'skiĭ, V. V., Design of mechanisms [Sintez mekhanizmov], Moscow-Leningrad, Gosud. Izdat. Tekh.-Teor. Lit., 1944, 387 pp.

A remarkably detailed and comprehensive presentation of one new and two classical methods for the design of cams and linkages. Part I (Artobolevskiĭ, 97 pp.) deals with cams and related mechanisms (Geneva stops, etc.). Part II (Dobrovol'skiĭ, 141 pp.) presents the Burmester theory of four-bar linkage design when a number (two, three, or four) of corresponding positions of certain members are given, or some similar conditions are prescribed. Part III (Blokh, 122 pp.), the most important one, introduces a method using complex numbers to specify the positions of the hinges, and applies it to 18 problems of four-bar linkage design and eight problems of five- and six-bar linkage design. The conditions combine a number of relative link positions with angular velocity data, and are more varied than the Burmester ones. Two more volumes, on computing and spatial linkages, are projected.
A. W. Wundheiler, USA

3009. Kobrinskiĭ, A. E., On the kinetostatic calculation of mechanisms with passive constraints and with play (in Russian), *Trudi Sem. teor. Mash. Mekh.* **5**, 20, 5–33, 1948.

If there is play in its kinematic pairs, a mechanism with passive constraints will be statically determinate. The author considers a double parallelogram linkage and an ellipsograph whose connecting rod is driven at its center by a crank. Because of backlash, the pair in which driving occurs will change, and such changes are accompanied by shocks. Using approximations of the first order in the clearances, the author determines the maximum reactions in the driving pairs and the velocities of the backlash motions during the change for several relations between the clearances. There is also some discussion of accelerations. The mathematics is simple and economical.
A. W. Wundheiler, USA

3010. Artobolevskiĭ, I. I., Zinov'ev, V. A., and Édel'shtein, B. V., A collection of problems on the theory of mechanisms and machines [Sbornik zadach po teorii mekhanizmov i mashin], Moscow-Leningrad, Gosud. Izdat. Tekh.-Teor. Lit., 1951, 195 pp.

3011. Egorov, V. V., A graphical method for the determination of the positions of spatial mechanisms (in Russian), *Trudi Sem. teor. Mash. Mekh.* **7**, 25, 5–68, 1949.

The author undertakes a fundamental classification of spatial (lower-pair) linkages, and begins, in Assur's tradition, with the classification of "dyads" [two bars, joined by a lower ("inner") pair, whose loose ends carry one ("outer") element of a pair each]. If the dyad becomes rigid when the loose ends are attached to fixed mating elements, it is an "Assur dyad" (of order zero). Because of the profusion of types of lower pairs (the author limits himself to eight), he obtains 27 Assur dyads. There are more if "floating" links (free to rotate about their own axis) are allowed. A code of designation is proposed. There is a tentative catalog of the important singular cases in which there is more freedom than the Grübler formula allows (e.g., three parallel sliding pairs), and their occurrence in mechanisms is exemplified.

If the positions of the two outer-pair elements are given, the position of the inner pair is determined as the intersection of two loci. This is the method of the paper. Procedures of descriptive geometry are used without any refinement. In view of the powerful method of dual numbers [see Dimentberg, F. M., AMR **5**, Rev. 19], the interest in graphical methods is on the wane (author makes no mention of Dimentberg's work). Questions of assembly and crank existence are discussed in rather general terms.
A. W. Wundheiler, USA

3012. Zinov'ev, V. A., Kinematic analysis of spatial four-bar mechanisms (in Russian), *Trudi Sem. teor. Mash. Mekh.* **7**, 28, 74–98, 1949.

The author reviews the elementary formulas of spatial point kinematics in spherical coordinates, and proceeds to apply the purely geometric ones to a linkage with two (adjacent) turning and two spherical pairs. Closure of the vector polygon of the links is the source of the equations. The method is straightforward, pedestrian, and nonspecific. Later, one turning pair is replaced by a helical one. The analysis is laborious, and no notice is taken of Dimentberg's work (reference in preceding review).
A. W. Wundheiler, USA

3013. Grodzinski, P., Application of eccentric gear mechanisms for obtaining variable angular velocities, *Machinery* **79**, 2020, 207–210, Aug. 1951.

Author describes series combinations of eccentric gearing and slider-crank mechanisms, explaining design procedure for obtaining output motions with special characteristics. Several examples give a good idea of the possibilities of such combinations.
A. S. Hall, USA

Gyroscopics, Governors, Servos

3014. Stein, T., The optimum regulation of hydraulic turbines (in German), *Schweiz. Bauztg.* **70**, 20, 287–292, May 1952.

Author investigates the optimum adjustment in the regulation of hydraulic turbines, and the accelerometric governor and the governor with dashpot are compared. If the latter is provided with regulating valve of great enough diameter, both kinds of governors are identical in function, but the dashpot governor is less sensitive on variations of self-regulation of the network. It is also more able to prolong its real time of closure, which appears necessary with regard to stability, if the flywheel effect should be reduced.

Reviewer believes that the combination of accelerometric governor with dashpot is the way to meet the said requirement [see Nechleba, M., *Houille blanche* **5**, 6, 808–811, Nov.-Dec. 1950].
Miroslav Nechleba, Czechoslovakia

3015. Dushkes, S. Z., and Cahn, S. L., Analysis of some hydraulic components used in regulators and servomechanisms, *Trans. ASME* **74**, 4, 595–601, May 1952.

See AMR **5**, Rev. 1298.

3016. Dubois-Violette, P.-L., Influence of heat transfer on the stability of temperature regulation (in French), *C. R. Acad. Sci. Paris* **233**, 1, 26–28, July 1951.

The phenomenon of cylindrical heat transmission introduces, in the expression for the ratio of the transmission $G(p)$ of a regulating servomechanism, a contribution which is reduced in the

majority of cases to a factor of the form $1/f(p)$, $f(p)$ being an integer function of p. Then the method of roots equalization [see Dubois-Violette, AMR 4, Rev. 1923] is applicable and permits the calculation of the limits of the stability of temperature regulation. From author's summary

3017. **Dubois-Violette, P.-L., Influence of heat transfer on the stability of temperature regulation** (in French), *C. R. Acad. Sci. Paris* **233**, 3, 232–234, July 1951.

Concrete examples show how to apply the method of roots equalization [see AMR **4**, Rev. 1923] in the case where the phenomena of heat transfer limit the possibilities of automatic regulation. Especially, results concerning crucible furnaces are determined. In a large measure they can be extended to other types of installation, without great changes.

From author's summary

3018. **Dubois-Violette, P.-L., Discussion of the stability of thermic regulations by the method of roots equalization** (in French), *C. R. Acad. Sci. Paris* **233**, 14, 730–732, Oct. 1951.

3019. **Brodin, J., Stability and parametric continuity of a linear servomechanism with time-dependent coefficients** (in French), *C. R. Acad. Sci. Paris* **234**, 8, 800–801, Feb. 1952.

Considering variable linear servomechanisms, the author gives a condition which is sufficient for stability, and also some formulas relating to the continuity of the error signal with respect to a parameter. The results, at least in part, are not essentially new, and their practical value appears to be slight.

Courtesy of Mathematical Reviews L. A. MacColl, USA

3020. **Boothroyd, A. R., and Westcott, J. H., The application of the electrolytic tank to servo-mechanism design,** "Automatic and manual control," New York, Academic Press, 87–102, 1952. $10.

Authors demonstrate use of electrically conducting sheet of uniform thickness for obtaining closed-loop characteristics of simple Ward-Leonard servomechanism with and without phase-advance stabilizing network in error channel. Method is based on the fact that an infinite conducting sheet with properly spaced electrical sources and sinks provides complex roots of polynomial equations from positions of saddle points (zero potential gradient in all directions) in the sheet. By logarithmic conformal transformation and by "matching," authors develop a practical finite electrolytic sheet having the properties of the infinite sheet and permitting investigation of 3 or 4 frequency decades. Using a novel procedure plotting lines of steepest descent from saddle points, roots at the assumed stability margin for the system (i.e., for the ratio of damping to angular frequency equal to 0.4) are determined. Accuracies of order of 1% are shown.

Walter W. Soroka, USA

Vibrations, Balancing

(See also Rev. 3247)

3021. **Heller, R., Transverse vibrations of uniform thin bars,** *J. acoust. Soc. Amer.* **24**, 3, 273–275, May 1952.

Continuing the theoretical work of Strehlke, Seebeck, and Lord Rayleigh, the transverse vibrations of uniform thin bars subject to the following end conditions are considered: (a) Both ends free; (b) both ends fixed; (c) one end free, the other fixed. Accurate values for the position of the nodes, antinodes, and points of inflection and of maximum curvature are calculated for the first few principal modes of vibration; several errors of long standing are rectified; and the amplitudes at the antinodes are computed. Approximate expressions, valid for the modes of higher order, are given for the node and antinode positions, as well as for the amplitudes at the antinodes.

From author's summary by Benjamin Smilg, USA

3022. **Spiegel, M. R., The random vibrations of a string,** *Quart. appl. Math.* **10**, 1, 25–33, Apr. 1952.

The similarity of the differential equations for an oscillating particle and for a linear electrical circuit is used to investigate random vibrations of a string by means of the theory developed for a system of such equations with special applications to noise in electrical networks. To use the theory for this special purpose, the string is assumed to be made up of $n + 2$ particles harmonically bound together, and Newton's law of motion is applied to each particle. By a limiting process, the results are carried over to a continuous string. Author arrives thus at a $2n$-dimensional Markoff process and is able to derive results coinciding with those obtained previously by G. E. Uhlenberg and G. A. Van Lear [*Phys. Rev.* **38**, 1583–1598, 1931]. The limiting process is not given in detail. Leif N. Persen, Norway

3023. **Hidaka, K., Vibration of a square plate clamped at four edges,** *Math. Japonicae* **2**, 97–101, 1951.

Aim of this paper is to calculate the approximate value of λ for the fundamental mode belonging to the boundary-value problem in which W satisfies the differential equation $\nabla^4 W - \lambda W = 0$ inside a unit square, and the boundary conditions $W = \partial W/\partial n = 0$ on the edges of the square. He replaces the differential equation by a difference equation containing values of W at the nodal points of square grid into which the original square is divided, and determines the lowest characteristic number for this difference equation. This is done for five different sizes of mesh, and then, in effect, the supposed true value is obtained by extrapolation from the five thus computed. The result secured is $\lambda = 13.2992\pi^4$ with an error which the author believes to be about 0.033%.

Courtesy of Mathematical Reviews W. E. Milne, USA

3024. **Collatz, L., On the numerical determination of periodic solutions in nonlinear vibrations** (in German), *ZAMP* **3**, 3, 193–205, May 1952.

Author presents approximate method for solving differential equations having nonlinear damping or nonlinear spring characteristics. Method involves expressing the solution in a form which can be solved by evaluating terms of Taylor's expansion. The necessary procedures are developed and two examples are presented. One example represents a two-degree-of-freedom system with linear damping and a nonlinear spring. The second example applies the procedure to a single-degree-of-freedom system with nonlinear damping and a linear spring.

W. J. Worley, USA

3025. **Gernet, M. M., Experimental determination of products of inertia, and dynamic balancing without a balancing machine** (in Russian), *Trudi Sem. teor. Mash. Mekh.* **9**, 33, 39–52, 1950.

The author proves that $I_{xy} = \frac{1}{2}(I_x + I_y) - I_{(xy)}$, where I_x, I_y, and $I_{(xy)}$ are the moments of inertia about the x-axis, y-axis, and their bisectant. I_{xy} is the product of inertia Σmxy. This reduces experimental measurements to six (three about the axes, and three about their bisectants). He then turns to the problem of dynamic balancing of a uniformly rotating statically balanced rotor. The (rotating) reactions in the bearings A, B form a couple of moment $\omega^2\sqrt{(I^2_{yz} + I^2_{zx})}$, z being the axis of rotation.

Since I_{yz}, I_{zx} can be determined by measurement, two positions of two balancing particles can be determined by computation alone. A discussion of the signs of their coordinates is necessary. A laboratory experiment is adduced with numerical data.

A. W. Wundheiler, USA

3026. Duncan, W. J., A critical examination of the representation of massive and elastic bodies by systems of rigid masses elastically connected, *Quart. J. Mech. appl. Math.* **5**, part 1, 97–108, Mar. 1952.

Author considers a beam of continuous mass and elasticity to be replaced by a system of rigid masses lying in the midpoints of equal segments while elastic properties remain unaltered, and he determines the difference (error) in the dynamical properties of the two systems. The resulting error in the natural frequency of any mode ultimately varies inversely as the square of the number of segments. When the masses do not lie in the midpoints of the sections, the error is much larger. As a rough rule, an error of 1% will not be exceeded if the equivalent system has at least thirteen masses in one wave length.

R. M. Rosenberg, USA

3027. Kampé de Fériet, J., Generalized harmonic analysis and some boundary-value problems (Lecture series no. 12, prepared by S. I. Pai), Univ. Maryland, Inst. Fluid Dynamics appl. Math., 103 pp., 1951–1952. $1.75.

Author introduces function class (p, a) as a generalization of Lebesgue class p. A major result is that classes $(2, a)$ with $a > 1$ all contain the physically significant class S of functions which have energy spectra, and that classes $(2, a)$ with $a \leqslant 1$ are all contained in S. A further result is that the class of a function is invariant under the application of certain types of integral transforms. In particular, it is shown that a solution of the heat equation is of class S at each instant if its initial value is a function of class S. A formula giving the energy spectrum of the solution in terms of the spectrum of the initial value is derived.

Author has provided a lucid explanation of parts of N. Wiener's theory, together with significant contributions of his own.

Clarence Ablow, USA

3028. Hansen, H. M., and Chenea, P. F., Mechanics of vibration, New York, John Wiley & Sons, Inc., 1952, xiii + 417 pp. $8.

Part I of book exhibits the characteristics of free, damped, and forced one-degree-of-freedom systems admitting of harmonic oscillatory analysis, by detailing both graphically and analytically the vectorial response of the individual spring, mass, and damper components. This emphasis on single elements is fundamental to a later development of the mobility method in which built-up systems are formed through proper vectorial combinations of elements. Paralleling the elemental development, the Newtonian equations are derived independently to give the student a broader perspective such that he may not become obscured in the detailed mechanics of the unit system. Thus, considerable duplication is introduced effecting a severe limitation on the coverage of engineering applications and practical problems. However, 346 well-chosen problems complete with answers cover the last 88 pages of the text and in a measure make amends and increase the scope of the work. For those seeking a solution to their own problem, the probability of finding a similar one here is good.

Part II deals with systems having multiple degrees of freedom. Prefacing it is a short 36-page chapter entitled "Classical methods," in which the engineer will find a fluent application of the Newtonian and Lagrangian methods, unusually complete and compact, without the use of matrix and tensorial calculus. Other topics include an example showing orthogonality of the principal modes, and the complex variable analysis of a damped vibration absorber. Part II is concluded by a chapter illustrating solution of the frequency equation by the Holzer and Graeffe methods.

Part III consists of a timely and well-measured introduction to elastic systems, transient vibrations, and nonlinear vibrations, the latter confined largely to solutions in elliptic functions. A chapter is devoted to each topic and illustrations include Rayleigh's method, relaxation frequency, indicial admittance, and impulsive response, together with Duhamel's integral for a general forcing function, all developed directly without reference to the Laplace transform, Fourier series, or boundary-value problem techniques.

Book may be characterized as a unique attempt to convey to the student the unifying experience and universal tool found in mathematical analysis together with some of the physical and numerical methods useful in engineering analysis of special types of problems. To this end, it is recommended to senior and first-year graduate students for whom it was written, and also to others, for its clarity and detail.

R. L. Leutzinger, USA

3029. Graffi, D., Forced oscillations for several nonlinear circuits, *Ann. Math. Princeton* **54**, 3, 262–271, Sept. 1951.

Consider the system $L_1\ddot{x}_1 + M\ddot{x}_2 + g_1'(x_1)\dot{x}_1 + x_1/C_1 = e_1(t)$; $M\ddot{x}_1 + L_2\ddot{x}_2 + g_2'(x_2)\dot{x}_2 + x_2/C_2 = e_2(t)$. Assume g_i' continuous, $g_i(0) = 0$, $\lim_{|x|=\infty} g_i(x)/x = R_i > 0$, e_i periodic of period T, $C_i > 0$, $L_i > 0$, $M^2 < L_1L_2$, $M^2(R_1C_1 + R_2C_2)^2 < 4R_1C_1R_2C_2L_1L_2$. Then a periodic solution of period T exists. The proof is based on Brouwer's fixed-point theorem.

Courtesy of Mathematical Reviews J. L. Massera, Uruguay

3030. Looney, C. T. G., Steady-state forced vibration of continuous frames, *Proc. Amer. Soc. civ. Engrs.* **78**, separ. no. 136, 21 pp., June 1952.

In this interesting paper, author considers technically important problem of steady-state damped vibration of continuous frames due to excitation by an oscillatory force having a frequency f very near one of the lower resonant frequencies. Method consists of following steps: (1) Assuming members to be simply supported, compute amplitudes of end slopes of loaded member; (2) for each member, compute magnitude of oscillatory end moment of frequency f which produces unit end slope, and compute corresponding slope at far end of member; (3) liquidate joint discontinuities assumed in step (1) by means of an iterative procedure using stiffness and carry-over factors of step (2). Author extends method to cases involving sidesway, demonstrates vectorial correction for concentrated mass, and presents examples.

Reviewer notes that formulas used by author in (1) and (2) are closely related to series expressions used by Sir Charles Inglis in 1934 [see also AMR **4**, Rev. 4374], and step (3) may be thought of in terms of mechanical admittances. It is noted also that expression quoted by author for response in general case as linear combination of natural modes erroneously indicates a common phase angle for all terms. This assumption may be sufficiently accurate only when the forcing frequency is nearly equal to one of the natural frequencies, in which case the corresponding mode will predominate. The paper, however, is limited later on to this case by the author.

John E. Goldberg, USA

3031. Lurie, H., Lateral vibrations as related to structural stability, *J. appl. Mech.* **19**, 2, 195–204, June 1952.

Author establishes a relation, approximately linear, between the buckling loads of plates and beams under edge thrust and the squares of the frequencies of the lateral vibrations. This rela-

tion is linear in cases where the modes of free vibrations are identical to the buckling modes. The significance of this approach is that the buckling load can be predicted by measuring the natural frequency ω^2 of free vibration first with no end load, then with an end load below the buckling load. A straight line between these two points intersects the load axis ($\omega^2 = 0$ axis) at the critical load. The test is nondestructive. Experiment shows discrepancies in case of plates. Various test results are given.
A. C. Eringen, USA

Wave Motion, Impact

(See also Revs. 2987, 3132, 3269)

3032. **Rakhmatulin, Kh. A., Transverse impact on a flexible filament by a body of given form** (in Russian), *Prikl. Mat. Mekh.* **16**, 1, 23–34, Jan./Feb. 1952.

A theoretical design of motion is adopted to solve the problem of transverse impact on a straight flexible filament of infinite length by a body of arbitrary form. An application of the general ideas is given in two special cases: (1) The striking body is a wedge, (2) the striking body is a circular cylinder. In the first case, elastic deformations are considered; it is shown that loss of energy occurs if the force discontinuity is passed and a plausible explanation of this phenomenon is given. Treating the transverse impact by a circular cylinder, author takes account of plastic deformations and shows how the problem of transverse impact could be solved by numerical methods. A table containing kinematographic analysis of the transverse impact by a wedge and by a circular cylinder on a rubber cord of 8-mm diam is added for justification of the theoretical design of motion.
J. Beránek, Czechoslovakia

3033. **Geleji, A., Duration and efficiency of the impact process in forging** (in German), *Acta Techn. Hung., Budapest* **1**, 3, 299–317, 1951.

A constant force, equal to the resistance to plastic flow of the part to be forged, decelerates the tup and accelerates the anvil. The forging period ends when the velocities of the tup and anvil become equal. The kinetic and elastic energy of the tup and anvil at this instant is the "loss" used in calculating efficiency of the forging process.

Elastic energy stored under impact in infinite foundation is also calculated, but the reviewer considers that the basic assumption used (equivalent to replacing the infinite solid by a long column having the same cross-sectional area as the forging) is unjustified [cf. displacements in semi-infinite solid under uniformly distributed load: Timoshenko and Goodier, "Theory of elasticity," p. 368].
G. Sved, Australia

3034. **Crook, A. W., A study of some impacts between metal bodies by a piezo-electric method,** *Proc. roy. Soc. Lond.* (*A*) **212**, 1110, 377–390, May 1952.

Experiments were performed in order to verify the Hertz-Saint Venant theory on impact, that is, the relation between the relative displacement and velocity after impact of the two bodies colliding and the force generated. The results agree closely with the theory, within the elastic range. In the plastic range, a "constant pressure" is found and an explanation of it is given on the basis of quasiviscous behavior. This explanation is not clear and gives the impression that, at the time of the paper, an interpretation of the relation between force and deformation under impact was still to be found. Piezoelectric crystals for the determination of stress were situated at the struck end of a lead bar, between such end and an anvil. This may appear necessary to avoid chattering of the crystal. However, the system may damp out the impact, increasing its duration and decreasing the maximum force, thus limiting the explored range of frequencies.
C. Riparbelli, USA

3035. **Davies, T. V., Gravity waves of finite amplitude. III. Steady, symmetrical, periodic waves in a channel of finite depth,** *Quart. appl. Math.* **10**, 1, 57–67, Apr. 1952.

Author studies steady, symmetrical, periodic waves in a channel of finite depth, using a method similar to the method explained for waves in a channel of infinite depth [see AMR **5**, Rev. 355]. Here also, author approximates the nonlinear boundary condition by another nonlinear condition and obtains a solution of first approximation; he also suggests a method for deriving higher approximations, but a demonstration of it is not given; solution seems only "fairly certain." Another point to be mentioned is that (author remarks) uniqueness of solution has not been solved. Nevertheless, author obtains a formula for velocity of propagation of the waves and for the breaking condition at the crest. He demonstrates, also, that the problem of the infinitely deep channel and the solitary wave are merely particular cases of given solution to the order of approximation in this paper.

Reviewer considers paper interesting; therefore, it would be important if author could demonstrate the convergence of higher approximations because, in this way, he would be certain that method approximates an irrotational system of waves; then demonstration of uniqueness of solution of first approximation would be useless.
Giulio Supino, Italy

Elasticity Theory

(See also Revs. 2985, 3017, 3044, 3049, 3055, 3099)

3036. **Teissier du Cros, F., On the fracture of a brittle prism along a longitudinal plane of symmetry** (in French), *C. R. Acad. Sci. Paris* **234**, 3, 296–297, Jan. 1952.

3037. **Legendre, R., Plane elastostatics** (in French), *Rech. aéro.* no. 27, 3–5, May/June 1952.

Any biharmonic solution can be written in terms of two independent functions of the complex variable z. L. Sobrero [*Mem. Atti. Accad. Italia* **6**, 1–64, 1934] has expressed biharmonic solutions with the help of a single hypercomplex function. It is shown that these difficult methods can be replaced by a simpler method involving the use of a single doubly complex function of the type $F(z + \epsilon\bar{z}, \epsilon)$, ϵ being a small parameter such that ϵ^2 and its higher powers can be neglected. The expansion of F to the first power of ϵ gives a biharmonic solution.

In cases when singularities like concentrated loads have to be considered, this method is found very suitable for practical applications. The case of a circular disk with concentrated loads at two points not on the same diameter is discussed in detail.
B. R. Seth, India

3038. **Mossakovskiĭ, V. I., On estimation of displacements in spatial contact problems** (in Russian), *Prikl. Mat. Mekh.* **15**, 5, 635–636, Sept./Oct. 1951.

When the distributed pressure p at a contact surface is known, to find the deformation and the resultant force and moments is a comparatively easy problem. The difficulty usually lies in finding p. However, from an estimated or known value of the surface deformation, the resultant forces and moments can be obtained from the formulas derived in this paper. The formulas are specialized to a contact surface of elliptic form. Author points out that the same formulas were obtained earlier by L. A. Galin in another way.
F. Niordson, Sweden

3039. Vaĭnberg, D. V., Method of discrete connections in a biharmonic contact problem for elastic bodies with circular symmetry (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **80**, 6, 865–866, Oct. 1951.

A thin ring, strengthening a hole in an infinite plate, is calculated approximately as a ring on elastic supports by substituting the continuous connection with connection at certain points.

Z. Bažant, Czechoslovakia

3040. Guyon, Y., Stresses near the ends of prismatic bodies which are loaded on their end surfaces (in French), *Publ. int. Assn. Bridge struct. Engng.* **11**, 165–226, 1951.

First part of paper deals with a half strip in plane strain, loaded along the end face by arbitrary forces. An approximate solution is obtained using Fourier series which are later expressed in finite terms. The approximation is refined for symmetrical normal loads by iteration. Extensive tables are given for the stresses under a single force of any position and inclination to the end face.

Second part of paper deals with rectangular prism loaded on its end face by symmetrically distributed normal forces. Approximate solution is obtained by use of double Fourier series. Numerical evaluation of results is given for a particular case.

W. T. Koiter, Holland

3041. Okubo, H., The stress distribution in a shaft press-fitted with a collar, *ZAMM* **32**, 6, 178–186, June 1952.

Author introduces stress functions satisfying the boundary conditions and the biharmonic equation. The derived expressions for radial stresses are in the form of a slowly convergent series and tedious to calculate. Making use of an approximate equation, author shows that the discrepancy between his exact and approximate solutions is about 2.2%.

Correcting factors to be used with the conventional formula are tabulated for various ratios of shaft and hub dimensions. To take into account the difference in materials, corresponding changes are presented.

Wilhelm Ornstein, USA

3042. Higuchi, M., Calculation of the stresses of the orthotropic strip with a hole, *Rep. Res. Inst. appl. Mech., Kyushu Univ.* **1**, 1, 33–45, Jan. 1952.

Solution is given in terms of stress functions for the plane stress problem of a strip of finite width and infinite length with different moduli in the transverse and longitudinal directions, and having a centrally located elliptical hole. Constants are determined for the strip subjected to longitudinal tension. Evaluation of stresses is made for strips of spruce and oak containing circular hole having diameter one half of the width of the strip. Results are compared with strips of isotropic material and indicate maximum stress is greater for the orthotropic case.

M. V. Barton, USA

3043. Nardini, R., On the energy dissipated by periodic forces through elastic hysteresis (in Italian), *R. C. Mat. Appl.* **10**, 3/4, 371–390, July/Dec. 1951.

Author considers an elastic body and supposes the forces acting on it, the strain and the stress, are periodic; he supposes also a lag of the stress behind the strain, therefore a dissipation of energy by hysteresis in the body. He gives a lower bound for the energy dissipated in a period and studies in detail the case in which only a couple with lever zero is acting on the body. He estimates, also, the numerical value of energy dissipated in the earth by the attraction of the moon; this value proves the reduction of earth's angular velocity.

Dario Graffi, Italy

Experimental Stress Analysis

(See also Revs. 3018, 3094)

3044. Frocht, M. M., Guernsey, R., Jr., and Landsberg, D., A photoelastic re-examination of notched tension bars, *J. appl. Mech.* **19**, 1, p. 124, Mar. 1952.

Tension bars with semicircular grooves have stress concentration factors between one and three. In 1937 Neuber published a calculation method based on an interpolation between theoretical limits. In 1947, Chih-Bing-Ling published similar factors derived from a theoretical solution. Authors re-examine the problem by means of the latest refinements of photoelastic measurements. The results give values of the stress-concentration factors, which lie in the neighborhood of the curve of Neuber but considerably below the curve of Ling. Therefore, authors believe that a re-examination of the mathematical solution of Ling is indicated.

H. Neuber, Germany

3045. Bergen, J. T., Analysis of localized stresses in drilled calender rolls, *Proc. Soc. exp. Stress Anal.* **9**, 2, 13–20, 1952.

Calender rolls used for rolling plastics often have drilled holes near the roller surface for circulation of a coolant. The holes cause stress concentration. A full-scale bakelite model of a section of a roller was caused to roll on a rubber wedge, and the stresses were calculated from the photoelastic fringe pattern. The force on the axle of the roller was also measured. To determine the stresses in the prototype, it was necessary to determine the force it exerted on its axle. Corresponding stresses on the prototype and full-scale model are in the same ratio as the forces on the axles per unit width of roll for the two cases.

Irwin Vigness, USA

3046. Bühler, H., and Schreiber, W., Measurement of residual stresses in bars and pipes by means of strain gages (in German), *ZVDI* **94**, 8, 216–218, Mar. 1952.

Authors recommend vacuum-drying of ERS gages when used in connection with Sachs boring-out method. Zero drift, which is of greatest importance in residual stress measurements, is not mentioned. Measurements carried out on one 150-mm diam steel bar show that, by using two gages in parallel and two pairs in series, round circumference average stresses can be determined.

R. Weck, England

3047. Sjöström, S., and Gunnert, R., Measurement of residual stresses (in Swedish), *Industrit. Norden* **79**, 23, 303–305, Nov. 1951.

Authors discuss different principles for measuring residual stresses in heat-treated or welded parts. Thin parallel layers of an initially flat test specimen are successively polished away, and the ensuing change of curvature is measured. From the assumption that the residual stresses are constant in surfaces parallel to the polished one, those stresses may be computed. This method by Stäblein and others is modified by Sjöström, who measures the strain with a strain gage on the surface parallel to the polished one instead of measuring the curvature.

To find residual stresses in a welded plate at some given point, Gunnert measures the distances between eight small conical cavities at the surface of the plate on a circle of about 9-mm diam. Then, a concentric annular groove is drilled outside the cavities to a depth of at least 6 mm. The material between the cavities is then released of stresses, and by measuring the new distances, one can find the initial residual stresses.

Frithiof I. Niordson, Sweden

3048. **Ramachandran, G. N., and Chandrasekharan, V., Photoelastic constants of sodium chlorate,** *Proc. Indian Acad. Sci.* (*A*) **33,** 3, 199–215, Mar. 1951.

From observations on crystals compressed along [100], [110], and [111] directions, $q_{11} - q_{12}$, $q_{11} - q_{13}$, and q_{44} were evaluated. All the four constants were independently obtained by combining these with polarization measurements of light diffracted by ultrasonic waves in the crystal. The values are: $q_{11} = 1.48$, $q_{12} = 3.88$, $q_{13} = 2.89$, $q_{44} = -1.58 \times 10^{-13}$ cm² dyne⁻¹; $p_{11} = 0.173$, $p_{12} = 0.258$, $p_{13} = 0.223$, $p_{44} = -0.0187$; q_{12} and q_{13} are different, as is to be expected from Bhagavantam's theory for crystal classes T and T_h. For stress along X-axis, the values of birefringence for observation along the Y- and Z-axes differ by as much as 70%, which is the largest observed so far for a cubic crystal.

From authors' summary

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 3041, 3060, 3067, 3080, 3086)

3049. **Donnell, L. H., Bending of rectangular beams,** *J. appl. Mech.* **19,** 1, p. 123, Mar. 1952.

A solution for the bending of rectangular beams under continuous normal load distribution acting on the top and the bottom surface is obtained as the sum of an elementary bending theory solution and corrective terms involving the prescribed normal load distribution, its derivatives, and the transverse coordinate of the beam. Those cases involving discontinuous load distribution may be solved by separate treatment of these discontinuities. The convergence of the present solution is better than that of the well-known Fourier solutions.

Conrad C. Wan, USA

3050. **Caquot, A., Note on the torsion of a straight beam built in at one end and free at the other** (in French), *Ann. Ponts Chauss.* **122,** 2, 127–130, Mar./Apr. 1952.

The effect of fixing one end of a thin-walled cylinder under torsion is found to diminish exponentially as the distance from the fixed end increases. This result is known, but no references are given even to the work of L. N. G. Filon, K. Wolf, or S. Timoshenko. The cylinder treated is one used by Esslinger (again no reference has been given), and it is pointed out that the difference between the numerical results calculated by her and those given by the classical theory is due to the fixing of the end section and not to the bending of the fibers.

B. R. Seth, India

3051. **Esslinger, M., Mlle., Torsion of curved I-beams** (in French), *Ann. Ponts Chauss.* **122,** 2, 131–149, Mar./Apr. 1952.

The differential equation is given which governs small flexural torsion of a circular I-beam. For a beam clamped at one side, free at the other side, and whose center line is a quarter of a circle, the following basic loading cases are worked out: (1) Constant torsional couple, constant shear force, and zero resultant bending couple in a normal plane; (2) pure torsional couple in the free normal plane, so in an arbitrary normal plane there is a torsional couple, bending couple, and zero shear force. The results of a numerical example coincide with experimental results. The flexural torsion reduces the deformations considerably.

W. L. Esmeijer, Holland

3052. **Wahl, A. M., The calculation of rectangular bar helical springs,** *J. appl. Mech.* **19,** 1, 119–122, Mar. 1952.

Helical springs of square or rectangular section are used where a large amount of energy must be stored in a small space. Author, a recognized authority, gives a concise presentation about these types of springs and their advantages and disadvantages. The given formulas and charts for calculating stress and deflection, taking the curvature effect into account, are very useful for the designer.

P. Kohn, Czechoslovakia

3053. **Allen, H. N. G., and Jones, T. P., Epicyclic gears,** *Trans. Inst. mar. Engrs.* **64,** 5, 79–99, May 1952.

Authors survey briefly some of the historical background of epicyclic gearing development and discuss the basic design principles of some well-tried forms likely to appeal to marine engineers. The importance of accurate manufacture to insure successful operation is stressed. The advantages of this type of gearing, including the savings of weight and space, are illustrated for a number of applications, and some comparisons are made between parallel shaft and epicyclic gears. Particular reference is made to the use of epicyclic gearing with marine auxiliaries.

From authors' summary

3054. **Tuplin, W. A., The form of tooth surfaces of creep-cut helical gears,** *Engineering* **172,** 4473, 4474, 4475, 4476; 484–486, 517–519, 550–552, 582; Oct., Nov. 1951.

Author considers in detail, by geometrical methods, tooth surfaces produced by hobbing with varying amounts of "creep" and concludes that (1) creep is desirable, i.e., error cycles per revolution of work table should not be an integer; (2) every rotating part has its own error cycle and, hence, creep fraction; (3) denominator of creep fraction should not be less than 30; (4) best overlap of cuts is obtained with creep fraction approximately $^1/_2$; fraction should therefore lie between 14/30 and 12/30; (5) variation in feed on noncreep machine leaves ridge pattern unaffected and roughness nearly unaltered; (6) creep finish and high spot distribution are materially affected by feed rate; effects of various feed rates, therefore, should be calculated beforehand to determine best compromise between cutting time and finish.

Ewen M'Ewen, England

3055. **Weinstein, A., On cracks and dislocations in shafts under torsion,** *Quart. appl. Math.* **10,** 1, 77–81, Apr. 1952.

Author extends the application of the theory of axially symmetric motion of a fictitious incompressible fluid in a space of five dimensions to the torsion problem of shafts of revolution, using the method of sources and sinks distributed outside the axis of symmetry. Theoretical formulas are derived for the stress concentration and displacements (1) in a flat interior crack, which is defined as a disk separating the material, with no stress across the surface, perpendicular to the axis of symmetry, and (2) in a dislocation, a surface of discontinuity for the displacements but with continuous stress across the surface, of the same size as the crack. The only admissible dislocation is, in this case, a rigid rotation of the negative side of the disk relative to its positive side. Author states that the method presented can be immediately applied to cavities of the form of oblate spheroids.

M. Kuipers, Holland

3056. **Chow, L., Conway, H. D., and Winter, G., Stresses in deep beams,** *Proc. Amer. Soc. civ. Engrs.* **78,** Separ. no. 127, 17 pp., May 1952.

Authors use method of a previous paper [AMR **5,** Rev. 1029] to calculate distribution and magnitude of bending and shear stresses. Results are presented graphically for beams simply supported at the lower edge having height-length ratios of $^1/_2$, 1, and 2. Loading conditions covered are: Distributed load, upper edge; distributed load, lower edge; concentrated load, center of upper edge; and concentrated load, center of lower edge. Concentrated loads and reactions are treated as if distributed over one

tenth of the beam's length. Case of loading at the third points of the upper edge is treated for height equal to length. Application to reinforced-concrete design is discussed.

C. M. Tyler, Jr., USA

Plates, Disks, Shells, Membranes

(See also Revs. 3037, 3042, 3068, 3069, 3088, 3104, 3109)

3057. Uflyand, Ya. S., Application of the Mellin transformation to the problem of bending of a thin elastic plate of wedge shape (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **84**, 3, 463–465, May 1952.

Deflection w of the plate is expressed in polar coordinates r, θ. The fourth-order partial differential equation for w is transformed into an ordinary one in θ by means of a Mellin transform. This ordinary differential equation is readily solved, and inversion of the transform yields solution w.

Author studies the special case of a concentrated force acting at the axis of symmetry on a simply supported plate, and expresses the solution w as a real integral, which is evaluated for the case of a right-angle wedge.

For the case when one wedge edge is simply supported and the other clamped, author computes the bending moment distribution along the clamped edge for a right-angle wedge. A similar result is obtained for a half-infinite plate where the edge $\theta = 0$ is clamped and $\theta = \pi$ is simply supported.

F. I. Niordson, Sweden

3058. Munakata, K., On the bending of a rectangular plate with four clamped edges, *Mem. Coll. Sci. Univ. Kyoto (A)* **26**, 1–8, 1950.

By means of a conformal mapping of the rectangle into the unit circle, author transforms the problem into one requiring the determination of two analytic functions in the circle. The main problem concerns the fulfillment of appropriate boundary conditions on the circumference of the circle. This is done by means of series developments in Legendre polynomials, leading to an infinite set of linear equations to determine the coefficients. The first four terms of the series are evaluated, and comparisons are made with results obtained by other methods.

J. J. Stoker, Jr., USA

3059. Nash, W. A., Several approximate analyses of the bending of a rectangular cantilever plate by uniform normal pressure, *J. appl. Mech.* **19**, 1, 33–36, Mar. 1952.

See AMR 5, Rev. 389.

3060. Kerkhof, W. P., The calculation of thick-walled cylindrical shells of carbon steel subject to internal pressure, *Ingenieur* **64**, 18, W.21–W.24, May 1952.

Author points out that existing design methods for thick-walled cylinders under internal pressure make use of Lamé's elastic theory and a permissible value of the maximum stress. Furthermore, such analysis does not reflect the true physical behavior of the cylinder with regard to yielding, excessive deformation, or rupture. He then presents formulas from various sources to more correctly describe (1) yielding at the inner surface of the cylinder, (2) deformation and rupture at temperatures below 650 F, (3) deformation and rupture at temperatures above 650 F. Finally, author concludes that the simple thin-walled cylinder formulas can be used with safety for thick-walled shells. The permissible stress is chosen from a tensile test or a creep test and modified by a factor of safety.

Reviewer is not convinced that author's recommendations present any improvement over existing design methods. However, article does bring to the attention of design engineers the fact that the common basis of design does not comply with the true physical behavior of cylinders under pressure.

M. C. Steele, Scotland

3061. Stippes, M., and Hausrath, A. H., Large deflections of circular plates, *J. appl. Mech.* **19**, 3, 287–292, Sept. 1952.

Author solves von Kármán's general equations for uniform load and simply supported edges by a perturbation method valid for small load parameters. Deflection and stresses are given in series form on assuming the perturbation method valid when load parameter is not small. Series converges slowly, whereas energy method of Way (1938) is fairly quick, but has advantage of showing physical interdependence of quantities more clearly and allows analysis of concentrated loads. Coefficients of series are tabulated.

K. H. Swainger, England

3062. Klebowski, Z., Application of the theory of a beam on an elastic foundation to certain structures (in French), *Bull. Acad. Polon. Sci. Lettres* **1**, 2, Suppl., 173–199, 1950.

The problem is to investigate the phenomena produced by a uniformly distributed circular-area load acting on an annular plate fixed at one end of a tube. The originality of the result obtained by author consists of a method which provides an easy way to take account of the finite length of the tube. On the basis of Winkler's theory of a beam on an elastic foundation, author considers as such a beam a longitudinal element of the tube of finite length, finding integration constants for the cases of the tube end loaded with uniformly distributed radial force P Kg/cm and bending moment M Kg cm/cm. Detailed tables giving the integration constants for tubes of various characteristics permit the results to be applied to the practical problem of finding P and M at the junction of tube and plate. The chief field of application of the results is the computation of flanges of various industrial vessels under pressure.

K. Zarankiewicz, Poland

3063. Salerno, V. L., and Levine, B., General stability of reinforced shells under hydrostatic pressure, Polyt. Inst. Brooklyn, Dept. aero. Engng. appl. Mech., *PIBAL Rep.* no. 189, 29 pp., Sept. 1951.

A strain-energy method of analysis is given of the problem of general instability (that is, instability over several bays) under uniform external pressure of a circular cylindrical shell reinforced by end bulkheads and by intermediate ring frames. It is assumed that negligible strain energy is stored in the end bulkheads and that the shell is there either simply supported or clamped. Approximations in the algebraic analysis are discussed, and an illustrative numerical example is worked.

H. G. Hopkins, USA

Buckling Problems

(See also Revs. 3031, 3063)

3064. Michielsen, H. F., The computation of flexural-torsional buckling loads, *J. appl. Mech.* **19**, 2, 214–219, June 1952.

The solution of the cubic equation given by Kappus for the torsional buckling load of a column often leads to insufficiently accurate results. To avoid the loss of accuracy, author has altered the equation by an ingenious series of transformations which finally yield an iterative form of solution. A thorough study has been made, and examples of the technique are given.

Reviewer recommends reference to Goodier, J. N. [*Cornell Univ. Engng. Exp. Sta. Bull.* nos. 27 and 28; Dec. 1941 and Jan. 1942]. Goodier has shown that the theory can be simplified if

the origin is taken at the shear center, thereby causing Kappus' terms R_x and R_y to vanish. R. B. McCalley, Jr., USA

3065. **Stowell, E. Z., Compressive strength of flanges,** *NACA Rep.* 1029, 14 pp., 1951.
See AMR **3,** Rev. 1888.

3066. **Hoff, N. J., Boley, B. A., Nardo, S. V., and Kaufman, Sara, Buckling of rigid-jointed plane trusses,** *Proc. Amer. Soc. civ. Engrs.* **76,** Separ. no. 24, June 1950 = *Trans. Amer. Soc. civ. Engrs.* **116,** 958–986, 1951.

Test results are presented on eight braced frameworks 45 to 60 in. long, 15 to 20 in. deep, and having 13 to 15 members. These show that convergence criterion of Hardy Cross moment-distribution method can be used with confidence to predict buckling loads if effect of gusset plates on stiffness and carry-over factors is taken into account. Computed buckling loads based on assumption that gusset-plate contribution to bending rigidity can be approximated by a hyperbolic law generally yield conservative results when the gusset-plate length is less than about $^1/_8$ of the length of the member to which it is attached. The assumption of infinite rigidity for gusset plate region gives unconservative results. Charts are presented for effect of gusset plates on stiffness and carry-over factors. Reasonable agreement between theory and experiment was found for two frameworks which buckled in the inelastic range. Nevertheless, further investigation of this problem is suggested by authors because large bending moments, whose magnitude could not be predicted theoretically but which influenced the effective modulus, were observed in the bars of the framework prior to buckling. An illuminating discussion of the convergence criterion is given. Samuel Levy, USA

3067. **Saibel, E., Buckling of continuous beams on elastic supports,** *J. Franklin Inst.* **253,** 6, 563–566, May 1952.

By minimizing the potential energy of the elastically supported continuous beam under axial compressive force, author shows that the equations and boundary conditions for the case of equal stiffness and equal spacing of the supports are identical to the equations and boundary conditions of the stretched vibrating cord carrying equal masses equally spaced. The known solution of the latter problem can be applied directly to determine the stiffness required to make the supports behave as though rigid. This closed solution is considerably simpler when compared to earlier exact and approximate solutions of Boobnov, Klitchieff, and Timoshenko. H. A. Lang, USA

3068. **Flügge, W., The optimum problem of the sandwich plate,** *J. appl. Mech.* **19,** 1, 104–108, Mar. 1952.

Paper considers the optimum design of flat sandwich plates by equating the over-all buckling and wrinkling stresses. The problem is to find the minimum weight for a given loading under the conditions that the dimensions of the loaded edge and the face modulus and density are fixed, with only the general type of core material specified, leaving the core density open within a prescribed range.

Paper is concerned mainly with the method of analysis, and no conclusions are drawn about specific materials or proper core densities for optimum design. George Gerard, USA

3069. **Plantema, F. J., Theory and experiments on the elastic over-all stability of flat sandwich plates,** Thesis, Techn. Hogesch. Delft, 67 pp., 1952.

Author uses method of partial deflections to obtain buckling loads and compares his method to those of Bijlaard, NACA, Forest Products Laboratory, and others, for many particular cases. In some cases, his method appears to be shorter than the others, but in other cases it does not appear to be too satisfactory (cannot handle all the boundary conditions). Author complains that published test data are too limited and do not consider the proper variables, whence he devotes half of his paper to the fabrication and testing of ten test specimens (in triplicate). Test results are 20 to 50% below calculated values, with large scatter in the three similar specimens of each type. Author argues that the initial deviations from flatness and weak regions of the core-to-face joints produced the low test loads.
B. E. Gatewood, USA

Joints and Joining Methods

(See also Rev. 3111)

3070. **Howard, D. M., and Smith, F. C., Fatigue and static tests of flush-riveted joints,** *NACA TN* 2709, 38 pp., June 1952.

Pertinent to the design of flush-riveted aluminum-alloy aircraft-joint details under repeated load, extensive tests showed superiority in both fatigue and static strength for the dimpling procedure as compared with machine countersinking. Considering joints using machine countersunk holes, comparison involving fatigue strength showed Alclad sheets better than bare sheets, lap joints better than butt joints. In both of the foregoing comparisons, no difference was noted with regard to static strength. Materials included 24S-T3 and 75S-T6 in both bare and Alclad thickness of 0.032 in. and 76S-T6 Alclad 0.064 in. thick. No satisfactory single relation between static properties and fatigue life covering the four materials could be found.
From authors' summary by B. G. Johnston, USA

3071. **Ball, J. G., and Cottrell, C. L. M., The weldability and mechanical properties of a series of low-alloy steels,** *J. Iron Steel Inst.* **169,** part 4, 321–336, Dec. 1951.

Difficulty in fillet-welding low-alloy structural steels arises from cracking due to excessive hardening in heat-affected zone. Twenty-seven experimental steels with constant carbon and molybdenum contents and varying contents of nickel, chromium, and manganese were investigated with respect to weldability by Reeve cracking test and with regard to mechanical properties by tensile test. Steels of 0.15% carbon and 0.25% molybdenum give best combination of mechanical strength after normalizing in $^1/_2$-in. square sections if alloyed by 0.8% of manganese, nickel, and chromium. Tempering at 600 C improves mechanical properties but reduces proof stress of more highly alloyed steels. Rutile electrodes were used throughout. Approximate safe limiting values for hardness are between 360 and 380 DPH.
R. Week, England

Structures

(See also Revs. 3030, 3062, 3066, 3261)

3072. **Bolton, A., A new approach to the elastic analysis of two-dimensional rigid frames,** *Struct. Engr.* **30,** 1, 1–13, Jan. 1952.

Author proposes a method for analysis of rigid frames in which he utilizes better features of slope deflection, moment distribution, and relaxation. Slope deflection is modified by use of type solutions; an example is given to illustrate reduction of number of equations. Characteristics of present methods used are: (1) Tabulation similar to that of moment distribution; (2) basic formula of slope deflection; (3) relaxation patterns; and (4) type-solution methods. Proposed solution has four steps: (1) Computation of constants with joints fixed against translation and

rotation; (2) computation of unit values for all possible deflections and rotations; (3) combination of results from steps two and three considering limiting conditions as imposed by type of structure; and (4) relaxation of fixed-end constants using patterns obtained in step three. Five examples are given which include single-span bents, multistory bents, multistory-three-span bents, and a gabled bent. This method permits one to use his experience in order to reduce amount of work and still use method of analysis that can be easily checked. C. M. Smith, USA

3073. Bažant, Zd., Mechanics of structures. Part IV [Stavebná mechanika] (in Czech), Praha, Vedecko-technicke nakladatelstvi, 1950, 242 pp.

Book deals circumstantially with the theory of beams and frames of nonuniform cross section. Its introductory chapters are concerned with the theory of the continuous beam and the full-walled arch without joints, both being of nonuniform cross section. Following parts show suitable methods for solving particular kinds of frameworks, which include both straight and curved rods. Included are the method of minimum deformation-work, the four-moment law, the method of force- and moment-distribution (Hardy Cross), and the deformation method. The concluding chapters contain methods for solving secondary stresses in frameworks.

Book includes many elaborated examples, which show in a practical way the efficiency of the methods used in particular cases, and enable the reader to understand and apply the theoretical conclusions directly to practical problems.

With this book, author concludes his systematic work on Mechanics of Structures, Parts I, II, III of which were edited in earlier years. Vladimir Kopřiva, Czechoslovakia

3074. Thomas, F. G., and Short, A., A laboratory investigation of some bridge-deck systems, *Proc. Instn. civ. Engrs.* **1**, 2, 125-172, Mar. 1952.

Authors describe theoretical (elastic) and experimental (both elastic and up to failure) investigations of composite bridge-deck systems at $^1/_3$ or $^1/_2$ scale. Systems studied are: Filler-joist (shallow rolled-steel sections entirely encased in a concrete slab); jack-arch, similar to above except that bottom of slab is not flat but arched between consecutive joists; slab and girder, with and without shear connections between slab and girders. Quantities studied were: Division of moment between slab and girders or joists; division of the total joist or girder moment among the several members; strains in steel members; crossties and shear connectors; and deflections. Authors conclude that there is more interaction among the several elements of a composite system than is commonly assumed in design; they propose that changes be made in specifications to take advantage of this.

M. P. White, USA

3075. Scruton, C., An experimental investigation of the aerodynamic stability of suspension bridges with special reference to the proposed Severn bridge, *Proc. Instn. civ. Engrs.* **1**, 2, 189-222, Mar. 1952.

Paper deals with investigations carried out at National Physical Laboratory on aerodynamic behavior of proposed Severn suspension bridge. A general historical note is followed by description of types of oscillations, and sectional and full-scale methods of model analysis. Results of analysis are also indicated and confirmatory conclusions drawn.

The bridge has a central span of 3200 ft, cable sag/span ratio of 1/11, and two side spans of 1000 ft. The suspended structure is of top-deck design with stiffening trusses 27 ft 6 in. deep and 78 ft apart. Investigations showed that reliable predictions of stability of any proposed design of truss-stiffened suspension bridges cannot, in general, be made without the aid of model tests in wind tunnels. The results of investigation can be used to guide the choice of a design, but to incorporate new features, confirmatory tests on stability are necessary.

A detailed study of suspension bridges stiffened by plate girders was not carried out, but general trends indicate that their over-all aerodynamic stability was comparatively low.

S. K. Ghaswala, India

3076. Morse, W., The analysis of fuselage frames, *Aircr. Engng.* **24**, 276, 277; 39-44, 49, 76-80, 88; Feb., Mar. 1952.

An easier way to calculate the static indeterminate quantities in fuselage frames is shown here, by taking the unknowns in the "elastic center," defined by $\bar{y} = \int (y/I)ds / \int (1/I)ds$ and $\bar{z} = \int (z/I)ds / \int (1/I)ds$. Four examples on built-in fuselage frames and complete fuselage rings are given.

M. Botman, Holland

3077. Huggenberger, A. U., Dam testing techniques. Measurement methods, instruments, and apparatus for testing of mass concrete structures [Talsperren-Messtechnik. Messverfahren, Instrumente und Apparate für die Prüfung der Bauwerke in Massenbeton], Berlin, Springer-Verlag, 1951, vii + 132 pp. DM 22.50.

The structural behavior of dams constructed of mass concrete has long attracted the attention of the engineering profession. Adequacy of designs can be determined only theoretically until such time as experimental data regarding structural behavior become available. Model experimentation alone has not been able to furnish the needed data. Investigation of existing structures must also be undertaken in order to provide complete data.

In this brief volume, author has collected descriptions of a large variety of measuring devices and has given an introduction to observational procedures, installation methods, and the handling of the instruments and apparatus. He believes that this is one of the great voids in structural behavior investigation, and he is to be congratulated on the way in which he has bridged this gap.

The tribute which the author pays to American progress will be appreciated by the American engineer who is acquainted with the literature on structural behavior available in this country, and who will at once recognize the large number of illustrations which have been included from this source, a fact which is inadequately brought out either in the text or in the bibliography. Omission of the customary credits may perhaps be excused by the author's footnote to the preface: "To protect the interests of others, no disclosure of the place and names of structures is made," but it would be of inestimable value to engineers in this country if a more extensive bibliography had been included.

Reviewer would like to point out that another void still remains, i.e., an adequate treatment of methods of analysis. To the best of his knowledge this entire field has not been satisfactorily treated in individual articles.

It is also noted that in some cases, particularly in the strain-meter field, much progress has been made both in design and in the analysis of results since the author's last visit to this country, and reviewer recommends reading recent American literature in place of this section of the book. In particular, the work at the Bureau of Reclamation on stress and strain has been documented in: (1) Raphael, J. M., "The determination of stress from measurements in dams," Third Congress on Large Dams, 1948; (2) Raphael, J. M., "The development of stresses in Shasta Dam," see AMR **5**, Rev. 2014; (3) Raphael, J. M., and Bruggeman, J. R., "Analysis of strain measurements in dams by use of punched card machines," Paper given at ASCE Convention, Denver, June 1952; (4) "Measurements of the structural be-

havior of Norris and Hiwassee Dams," *Tech. Monogr.* no. 67, Tennessee Valley Authority, Knoxville, Tenn., Aug. 1950.

J. R. Bruggeman, USA

3078. Craemer, H., Maximum condition, approximation, and superposition in theories of plasticity and of earth-pressure, *Proc. Instn. civ. Engrs.* **1, 2,** 243–244, Mar. 1952.

Abstract of a paper held in the Institution of Civil Engineers Library. Author reviews some limit-analysis theory applied to beams and plates, and notes the fact that a large error in locating the yield hinges produces only a small error in the collapse load. A similar hypothesis is advanced for soil mechanics, as to the effect of the location of the sliding surfaces. "In a rather complex earth-pressure problem, the author has found an error of only 7% by adopting a sliding plane inclined at 60° instead of 52.5° as obtained by a maximum condition."

Philip G. Hodge, Jr., USA

3079. Gyengö, M. T., Tests for investigation of the fracture state of beams in reinforced concrete (in French), *Acta Techn. Hung., Budapest* **2,** 2/4, 345–368, 1952.

Tests was made on 60 reinforced-concrete beams to determine the reliability of the ultimate design procedure of the new Hungarian specifications. Spans, dimensions, and cross-sectional shapes varied. Principal variables were strength of concrete and type of reinforcement. Three types of plain round bars were used: 12-mm diam with a flat yield point at 60,000 psi; 8.3-mm diam with no yield point but with a break in the curve at about 86,000 psi; and a cold-drawn 5-mm wire with an ultimate strength of 222,000 psi. Steel percentages varied from 0.17 to 1.66%. Concrete strengths varied from 1165 to 5700 psi for 8-in. cubes. Tests were made on simple span with symmetrical two-point loading. Observations included ultimate load, manner of failure, and formation of cracks. Some tests were made with 25 repetitions of load-producing stress of 64,000 psi in steel. Only 37 beams failed in flexure, the others failed in shear or bond. Test results for flexural failures compared with predictions of theory with fair agreement. No study was made of shear or bond failures.

C. P. Siess, USA

3080. Jäger, K., The most probable moment distribution in statically indeterminate reinforced-concrete beams (in German), Alfons Leon Gedenkschrift, Verlag Allg. Bau-Z., Wien, 40–48, 1952.

After reviewing briefly the contradiction existing between the assumptions, as usually made, for the dimensioning (Stage II) and computation of the internal forces (Stage I) in statically indeterminate reinforced-concrete structures, author shows that, actually, the moment distribution in statically indeterminate reinforced-concrete beams depends upon the "deformation capacity" (Formänderungsfähigkeit), i.e., upon both the concrete cross-section dimensions and the steel reinforcement's area and distribution. This makes necessary the determination of bending moments at the "critical stage" (kritische Zustand), for only in this way does the required safety factor seem to be secured.

Based on the stress-strain functions developed in his previous book ["Festigkeitsnachweis im Stahlbetonbau," Wien, 1948], author presents a detailed study of moment distribution for two types of beams: (1) Built in at one end and supported at the other; (2) with both ends built in. For each case, to obtain the influence of reinforcement, study is made of two extreme conditions: (a) Steel reinforcement theoretically adapted exactly to the moment distribution; (b) constant areas of steel reinforcement at, respectively, the negative and the positive regions of moments, with values adapted to the maximum bending moments in each region. The real condition of discontinuous variation of reinforcement area is also considered. In each case, the moment distribution is computed for both the "critical" and the actual stage of loading. The limit of error is given for the method usually adopted.

From the various conclusions drawn, reviewer remarks: (1) The moment distribution depends mainly upon the steel area at the sections where the moments are maxima; (2) in case of complete utilization of steel reinforcement at the sections with greatest moments, there is no appreciable difference between moment distribution at the critical and actual stages of loading; (3) the most probable distribution may be obtained considering a constant area of steel reinforcement at, respectively, the negative and positive regions of moments, with values adapted to the maximum bending moments in each region; (4) the error made by computing with the usual method (homogeneous beam) implies a reduction of the safety factor, which in certain cases attains 12%. Author considers this value of the error too small to be experimentally confirmed, which would explain, as he says, the conclusions drawn by Prof. Mörsch ["Statik der Gewölbe und Rahmen," Teil A, K. Wittmer, Stuttgart, 1947].

Reviewer notes especially author's statement that it is impossible to consider, in reinforced concrete, the action of so-called "plastic hinges" (Flies-gelenken), with the consequent readjustment of moment distribution; the reinforced-concrete beam would show all the characteristics of brittle rupture. Does author refer, for instance, to the work done by K. W. Johansen ["Brudlinieteorier," Jul. Gjellerups Forlag, Koebenhavn, 1943]?

Ivo Wolff, Brazil

3081. Olszak, W., Prestressed concrete and reinforced concrete; two different ideas (in Polish), *Inzyn. Budown.* **6,** 10/12, 675–690.

A very clear distinction is emphasized between the idea of prestressed concrete and (ordinary) reinforced concrete. It is shown that prestressing is independent of reinforced concrete in principle as well as in practical application. Prestressing may be obtained without any steel reinforcement, and consequently its execution may be quite different (e.g., it may be put outside the element).

Mechanism of concrete deformations is discussed; effect of its "adaptation" to exterior mechanical conditions (overloading) is shown. Advantages of prestressing are emphasized, the best results being obtained for members acted on by tensile forces.

J. Nowiński, Poland

3082. Abeles, P. W., Further notes on the principles and design of prestressed concrete, *Civ. Engng., Lond.* **45, 46, 47;** 529, 530, 531, 532, 533, 535, 537, 538, 540, 541, 544, 545, 549, 550; 443–445, 508–512, 579–581, 657–662, 728–731, 38–40, 187–190, 262–264, 439–442, 524–527, 775–778, 860–864, 234–237, 318–321; July, Aug., Sept., Oct., Nov., 1950; Jan., Mar., Apr., June, July, Oct., Nov., 1951; Mar., Apr. 1952.

The following topics are discussed in this series of articles on prestressed-concrete beams: Behavior between cracking and failure, bond, ultimate resistance, factor of safety and load factor, different views on some principles, types of structures, cracking in reinforced and prestressed concrete, suitability of cross section, deflections, bending of post-tensioned cables, shear reinforcement, composite sections, statically indeterminate structures, First Report on Prestressed Concrete (British), some developments in research since 1948, and some new developments in methods and systems of prestressing. Individual topics are discussed briefly and the discussion is supported by experimental evidence and numerous examples of design.

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rotation; (2) computation of unit values for all possible deflections and rotations; (3) combination of results from steps two and three considering limiting conditions as imposed by type of structure; and (4) relaxation of fixed-end constants using patterns obtained in step three. Five examples are given which include single-span bents, multistory bents, multistory-three-span bents, and a gabled bent. This method permits one to use his experience in order to reduce amount of work and still use method of analysis that can be easily checked. C. M. Smith, USA

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J. R. Bruggeman, USA

3078. **Craemer, H., Maximum condition, approximation, and superposition in theories of plasticity and of earth-pressure,** *Proc. Instn. civ. Engrs.* **1**, 2, 243–244, Mar. 1952.

Abstract of a paper held in the Institution of Civil Engineers Library. Author reviews some limit-analysis theory applied to beams and plates, and notes the fact that a large error in locating the yield hinges produces only a small error in the collapse load. A similar hypothesis is advanced for soil mechanics, as to the effect of the location of the sliding surfaces. "In a rather complex earth-pressure problem, the author has found an error of only 7% by adopting a sliding plane inclined at 60° instead of 52.5° as obtained by a maximum condition."

Philip G. Hodge, Jr., USA

3079. **Gyengö, M. T., Tests for investigation of the fracture state of beams in reinforced concrete** (in French), *Acta Techn. Hung., Budapest* **2**, 2/4, 345–368, 1952.

Tests was made on 60 reinforced-concrete beams to determine the reliability of the ultimate design procedure of the new Hungarian specifications. Spans, dimensions, and cross-sectional shapes varied. Principal variables were strength of concrete and type of reinforcement. Three types of plain round bars were used: 12-mm diam with a flat yield point at 60,000 psi; 8.3-mm diam with no yield point but with a break in the curve at about 86,000 psi; and a cold-drawn 5-mm wire with an ultimate strength of 222,000 psi. Steel percentages varied from 0.17 to 1.66%. Concrete strengths varied from 1165 to 5700 psi for 8-in. cubes. Tests were made on simple span with symmetrical two-point loading. Observations included ultimate load, manner of failure, and formation of cracks. Some tests were made with 25 repetitions of load-producing stress of 64,000 psi in steel. Only 37 beams failed in flexure, the others failed in shear or bond. Test results for flexural failures compared with predictions of theory with fair agreement. No study was made of shear or bond failures.

C. P. Siess, USA

3080. **Jäger, K., The most probable moment distribution in statically indeterminate reinforced-concrete beams** (in German), Alfons Leon Gedenkschrift, Verlag Allg. Bau-Z., Wien, 40–48, 1952.

After reviewing briefly the contradiction existing between the assumptions, as usually made, for the dimensioning (Stage II) and computation of the internal forces (Stage I) in statically indeterminate reinforced-concrete structures, author shows that, actually, the moment distribution in statically indeterminate reinforced-concrete beams depends upon the "deformation capacity" (Formänderungsfähigkeit), i.e., upon both the concrete cross-section dimensions and the steel reinforcement's area and distribution. This makes necessary the determination of bending moments at the "critical stage" (kritische Zustand), for only in this way does the required safety factor seem to be secured.

Based on the stress-strain functions developed in his previous book ["Festigkeitsnachweis im Stahlbetonbau," Wien, 1948], author presents a detailed study of moment distribution for two types of beams: (1) Built in at one end and supported at the other; (2) with both ends built in. For each case, to obtain the influence of reinforcement, study is made of two extreme conditions: (a) Steel reinforcement theoretically adapted exactly to the moment distribution; (b) constant areas of steel reinforcement at, respectively, the negative and the positive regions of moments, with values adapted to the maximum bending moments in each region. The real condition of discontinuous variation of reinforcement area is also considered. In each case, the moment distribution is computed for both the "critical" and the actual stage of loading. The limit of error is given for the method usually adopted.

From the various conclusions drawn, reviewer remarks: (1) The moment distribution depends mainly upon the steel area at the sections where the moments are maxima; (2) in case of complete utilization of steel reinforcement at the sections with greatest moments, there is no appreciable difference between moment distribution at the critical and actual stages of loading; (3) the most probable distribution may be obtained considering a constant area of steel reinforcement at, respectively, the negative and positive regions of moments, with values adapted to the maximum bending moments in each region; (4) the error made by computing with the usual method (homogeneous beam) implies a reduction of the safety factor, which in certain cases attains 12%. Author considers this value of the error too small to be experimentally confirmed, which would explain, as he says, the conclusions drawn by Prof. Mörsch ["Statik der Gewölbe und Rahmen," Teil A, K. Wittmer, Stuttgart, 1947].

Reviewer notes especially author's statement that it is impossible to consider, in reinforced concrete, the action of so-called "plastic hinges" (Flies-gelenken), with the consequent readjustment of moment distribution; the reinforced-concrete beam would show all the characteristics of brittle rupture. Does author refer, for instance, to the work done by K. W. Johansen ["Brudlinieteorier," Jul. Gjellerups Forlag, Koebenhavn, 1943]?

Ivo Wolff, Brazil

3081. **Olszak, W., Prestressed concrete and reinforced concrete; two different ideas** (in Polish), *Inzyn. Budown.* **6**, 10/12, 675–690.

A very clear distinction is emphasized between the idea of prestressed concrete and (ordinary) reinforced concrete. It is shown that prestressing is independent of reinforced concrete in principle as well as in practical application. Prestressing may be obtained without any steel reinforcement, and consequently its execution may be quite different (e.g., it may be put outside the element).

Mechanism of concrete deformations is discussed; effect of its "adaptation" to exterior mechanical conditions (overloading) is shown. Advantages of prestressing are emphasized, the best results being obtained for members acted on by tensile forces.

J. Nowiński, Poland

3082. **Abeles, P. W., Further notes on the principles and design of prestressed concrete,** *Civ. Engng., Lond.* **45, 46, 47**; 529, 530, 531, 532, 533, 535, 537, 538, 540, 541, 544, 545, 549, 550; 443–445, 508–512, 579–581, 657–662, 728–731, 38–40, 187–190, 262–264, 439–442, 524–527, 775–778, 860–864, 234–237, 318–321; July, Aug., Sept., Oct., Nov., 1950; Jan., Mar., Apr., June, July, Oct., Nov., 1951; Mar., Apr. 1952.

The following topics are discussed in this series of articles on prestressed-concrete beams: Behavior between cracking and failure, bond, ultimate resistance, factor of safety and load factor, different views on some principles, types of structures, cracking in reinforced and prestressed concrete, suitability of cross section, deflections, bending of post-tensioned cables, shear reinforcement, composite sections, statically indeterminate structures, First Report on Prestressed Concrete (British), some developments in research since 1948, and some new developments in methods and systems of prestressing. Individual topics are discussed briefly and the discussion is supported by experimental evidence and numerous examples of design.

Large portion of the paper is devoted to the discussion of the

behavior of prestressed-concrete beams. Author points out that from a practical point of view the principal characteristics of such beams are the factor of safety against cracking and the resilience. Beams with a large factor of safety against cracking have small resilience, and vice versa. Depending on these two characteristics, four types of beams may be distinguished: (1) Those with a high degree of safety against cracking, but with a relatively small resilience; (2) those which do not crack under the service loads, but have a small factor of safety against cracking; (3) those which crack under occasional high loads, but with cracks invisible under normal service conditions; (4) those with visible fine cracks comparable to the ordinary reinforced concrete; this type of beam has the largest resilience. Which of the four types should be selected depends on the purposes of the structure.

Design formulas are given for computing the ultimate resistance, the working load stresses, and deflections for beams of various cross sections. Ultimate formulas are given only for under-reinforced bonded beams, and it is pointed out that the present knowledge does not permit computing the ultimate resistance of over-reinforced and of unbonded beams. Directives are given for the design of shear reinforcement. The use of formulas is illustrated by numerical examples.

Paper brings the reader up to date. It gives a clear picture of the behavior of prestressed-concrete beams, even though the clarity of some parts—especially of those dealing with research findings—suffers from brevity. In addition, author points out the gaps in the present knowledge of the subject.

I. M. Viest, USA

3083. Schwarz, R., Contribution to the calculation of creep losses in prestressed structural members in reinforced concrete (in German), *Bauingenieur* **27**, 3, 85–90, Mar. 1952.

This study, based on author's earlier study of creep in ordinary reinforced-concrete members [*Beton und Eisen*, **38**, 185–190 and 202–205], deals with creep losses of prestress in pretensioned, and bonded and unbonded posttensioned structural members. Basic differences between these three types of prestressed concrete members are defined, and an expression is derived for computing the loss of prestress due to the elastic shortening of pretensioned members.

The creep losses are divided into three categories: Those due to dead load applied simultaneously with the prestress; those due to dead load applied after prestressing; and those due to shrinkage. Theoretical expressions are derived for each category of losses, and the final loss of prestress is defined as the sum of the individual losses.

The use of the theoretical expressions is demonstrated with a numerical example.

I. M. Viest, USA

⊗3084. Komendant, A. E., Prestressed concrete structures, New York, McGraw-Hill Book Co., Inc., 1952, xiv + 261 pp. $6.

Book quite adequately covers the whole field of prestressed concrete. The first chapter, covering the various techniques of prestressing and their ramifications, and the second chapter, describing the special quality steel and concrete, make very easy reading and yield a great deal of information. The section on design requires a thorough grounding in hyperstatic stress computations, but sample calculations clarify the entire chapter. Reviewer feels that the book would have gained by placing the study of friction and anchorage of cables and the design of a watertight concrete mix at the end of the chapter on materials, instead of at the end of the chapter on design. Book concludes with a detailed description of numerous existing structures which give the reader a keener appreciation of the whole problem.

Robert Quintal, Canada

3085. Worley, H. E., Triaxial design correlated with flexible pavement performance in Kansas, "Triaxial testing soils bitum. mixtures," *ASTM Spec. tech. Publ.* no. 106, 112–137, 1951. $3.50.

Paper is intended to cover thoroughly the application of triaxial testing to flexible pavement design. Methods and procedures of sampling are not described in detail except as necessary in showing preparation for testing. A comprehensive discussion of test data and field performance is included. A less detailed explanation is given for the use of test data in regard to foundations, slopes, evaluation of materials for bases and surfaces, and subgrades for rigid pavements.

The required thicknesses of base course and wearing surface, as determined by this method of test procedure and design, consistently parallel the thicknesses which have given satisfactory service for several years.

From author's summary

3086. Esslinger, Maria, Analysis of a tubular arch (in French), *Ossature métall.* **17**, 2, 97–104, Feb. 1952.

Equations are derived for the analysis of a tubular arch of constant radius. The arch is assumed fixed at the supports. It is shown that the variations in the moment of inertia due to deformation have a secondary effect on the stresses. The theoretical development considers stresses due to symmetrical, unsymmetrical, and wind loads and temperature.

Elio D'Appolonia, USA

3087. Rowe, R. S., and Shore, S., Model arches in the flexible range. Testing technique, *Proc. Soc. exp. Stress Anal.* 9, 2, 31–42, 1952.

In a flexible arch, displacements due to loads cannot be neglected as is customary in the analysis by elastic theory. Paper presents the methods and test equipment used to measure the moments and thrusts in flexible model arches. The methods described are satisfactory for the model arches and applicable to other static tests. Circular arches of varying ratios of rise to span were tested. A very complete SR-4 strain-gage equipment has been used. The measured strains are indicated in a data sheet in a very good way. The stability of the gages was found to be excellent by spot-checking gage readings under a variety of conditions of temperature and humidity. In general, the SR-4 strain-gage equipment proved to be extremely satisfactory, the inherent inaccuracy of the gages amounting to ±1%. The observed values are compared with values computed by the theory of elasticity.

Ludwig Föppl, Germany

3088. Watts, G. W., and Lang, H. A., Stresses in a pressure vessel with a conical head, *Trans. ASME* **74**, 3, 315–324, Apr. 1952.

Determination of stresses at the connection of a cylindrical with a conical shell requires tedious calculation, even if both shells are "long," with only four arbitrary constants in all, as in the paper considered, in which the cone is used as head (excluding, e.g., cylindrical stack on conical shell). In problems of this kind, it is always desirable to have all necessary computations carried out once and for all and for a wide variety of all parameters influencing the results. True, design of the combined shell is not dominated only by theory of thin shells and not even by elasticity alone, but solution of the standard elastic shell problem is the foundation of the design problem. Author has studied this foundation in an expertly truly exemplary way and in a practically inexhaustive manner. Notwithstanding the great amount of labor involved, the solution is given—in form of stress indexes taking care of shear, direct stress, and bending—for varying values of apex angle, ratio of thicknesses of cylinder and cone, and ratio of diameter to cone thickness. The abundance of numerical tables

and graphs will be deeply appreciated by designers, who, in most cases, can derive the solution they need by means of interpolation from the tables given. Publication of tables of this kind is a highly meritorious achievement. I. Malkin, USA

3089. Kochina, I. N., and Polubarinova-Kochina, P. Ya., On the application of smooth contours to the foundations of hydraulic structures (in Russian), *Prikl. Mat. Mekh.* **16,** 1, 57–66, Jan./Feb. 1952.

In investigations of the speed of the flow of water through a porous medium along the contours of the foundations of hydraulic structures whose cross sections consist of straight line segments, it is found that at the corners there can exist very large velocities of flow. Such high velocities at any part of the foundation embedded and floating on a porous medium are very undesirable, since their presence can cause deformations in the supporting medium and thus endanger the stability of the structure. When the supporting porous medium is assumed to be of infinite depth, it has been previously shown that a contour for the cross section of the support in the form of a semicircle was the best shape in so far as the distribution of the speeds of flow of the water along the support is concerned, for along it the speed is constant. The present paper considers the problem of determining the most desirable smooth contours for the supports of hydraulic structures floating in a porous medium of finite depth. The most desirable contours are here taken as those along which the rate of flow is constant. The distributions of velocities along supports whose cross sections are rectangles with rounded corners and certain other cases are also considered.

Courtesy of Mathematical Reviews H. P. Thielman, USA

3090. Chang, F. K., and Johnston, B. G., Torsion of plate girders, *Proc. Amer. Soc. civ. Engrs.* **78,** Separ. no. 125, 46 pp., Apr. 1952.

Both analytical and experimental investigations of torsion of built-up members have been lacking. Approximate formulas are derived for rivet pitch and weld-spacing to develop integral action in I-section girders. Tentative relations are given for torsional constant when rivet pitch exceeds value necessary for integral action. Tests of large-scale specimens show reasonable agreement with theory and demonstrate superiority of welded girders. Torque-twist curves deviate from linear behavior anywhere from 32 to 72% of theoretical yield moment for riveted and bolted sections. Behavior is governed mainly by bolt tension, method of driving rivets, and rivet or bolt location and pitch. Typographical errors occur in Fig. 6 (c), first part of equation (17a), and third term in right-hand member of equation (26). Robert B. McCalley, Jr., USA

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 3043, 3078, 3083, 3109, 3110, 3119, 3122)

3091. Andrade, E. N. da C., The flow of metals, *Engineering* **173,** 4502, 601–602, May 1952 = *Engineer* **193,** 5025, 5026; 658–659, 692–693, May 1952 = *J. Iron Steel Inst.* **171,** part 3, 217–228, July 1952.

Author sets forth a formula to define the creep of metals under constant stress. Total creep is defined as the result of transient creep, caused by slip within the crystal grains, and permanent creep, caused by the rearrangement of the grains. The author claims that his short-time tests, as well as the 1000-hr test of L. M. T. Hopkin, agree very well with the proposed formula.

Two other facts peculiar to creep are discussed. Creep in tension bears no relation to creep under other conditions of stress. And finally, the total creep under intermittent loading is greater than the creep under uninterrupted loading for the same elapsed time. V. P. Zimnoch, USA

3092. Kochendörfer, A., Glide phenomenon and strain-hardening in metallic materials (in German), *ZVDI* **94,** 10, 267–273, Apr. 1952.

The purposes and methodology of physics and technological mechanics are discussed with reference to slippage and strain-hardening of metals, and the differences in behavior of single crystals and multicrystalline systems are exposed. Models are shown for: Slippage in a hexagonal crystal; homogeneous shear deformation in an ideal crystal; force relationships in homogeneous and nonhomogeneous shear deformation; and elongation of multicrystalline systems.

Graphs are given for: Stress-slippage relationships of Ni, Cu, Ag, Al, Au, Mg, Zn, Sn(1), Sn(2), and Cd, at room temperature; relation of critical shear stress and strain-hardening to temperature; stress-strain relationship for hexagonal, space-centered cubic, and face-centered cubic metals; and tensile strength of Cu and Al as function of rate of elongation for different temperatures. Application of crystal physics to technical problems is discussed. Hans F. Winterkorn, USA

3093. Kuhlmann-Wilsdorf, D., van der Merke, J. H., and Wilsdorf, H., Elementary structure and slip band formation in aluminum, *Phil. Mag.* (7) **43,** 341, 632–644, June 1952.

Theoretical calculation of the minimum spacing at which slip lines should be found agrees with the elementary spacing observed by electron microscope. Calculation is based on idealized model in which uniform rows of dislocations of opposite sign move past each other. Minimum slip spacing is taken to be that at which the interaction between rows is just insufficient to prevent their relative motions.

Several experiments that indicate that the fine elementary slip process is confined to the surface are discussed, and, though not explicitly stated, reviewer senses that the generation of slip in the surface region is favored by the authors.

A mechanism for annihilation of most of the elementary slip by the formation of dislocation loop pairs is described. Only the stronger dislocations survive the process to become slip bands in regions far from their point of origin. A. N. Holden, USA

3094. Braun, I., and Reiner, M., Problems of cross-viscosity, *Quart. J. Mech. appl. Math.* **5,** part 1, 42–53, Mar. 1952.

Reiner's equation of a general viscous liquid, which differs from Stokes' equation by a quadratic term defining a coefficient of cross viscosity, is integrated for the cases of (1) a cylinder rotating in a liquid, (2) the rotating coaxial cylinder viscometer, (3) the tube viscometer, (4) the parallel plate torsional viscometer. This provides criteria for crucial experiments to decide whether the centripetal effect in certain liquids is totally or partially due to cross viscosity. Provided the liquid climbs up the cylinder in (1), there should be radial tension upon the internal cylinder of (2). In (3), the wall pressure should be greater than the Newtonian hydrodynamic pressure by a term involving the ordinary shear viscosity. Finally, in (4) there should be a parabolically distributed pressure upon the plate, which vanishes at a distance from the axis of rotation equal to $1/(3)^{1/2}$ times the radius of the plate.

From authors' summary by C. F. Bonilla, USA

3095. Fraenkel, G. K., Visco-elastic effect in solutions of simple particles, *J. chem. Phys.* **20,** 4, 642–647, Apr. 1952.

Calculations performed on a simple model—a solution contain-

ing elastic dumbbells, i.e., particles consisting of two rigid bodies held together by elastic forces—show that dilute solutions of asymmetric particles possess a frequency-dependent intrinsic viscosity and a rigidity. Part of the rigidity would arise from inelastic dumbbells and caused by forces, usually neglected as a result of Brownian motion. An additional rigidity is caused by the elastic nature of the dumbbells. At sufficiently low or high frequencies, the results approximate a form similar to the ordinary dispersion formula for the dielectric constants of solutions and involve two relaxation times, one at low frequencies and one at high frequencies. The results for inelastic dumbbells are described by one relaxation time and follow the usual dispersion curve for all frequencies.

In the calculation, the Brownian-motion distribution function describing the probability of a dumbbell being at a given position with a given orientation is first determined. This distribution function is used to calculate the contribution of the Brownian motion to the forces on the particles. From these forces the stresses added to the solution by the particles are determined, and the incremental viscosity is calculated from these stresses.

S. G. Ward, England

3096. Fromm, H., and Hartung, W., Measurement of the shear modulus of viscous liquids (in German), *Technik* **7**, 3, 126–130, Mar. 1952.

Paper under review is based on papers by Pieruschka [AMR **4**, Revs. 3550, 3927] and by H. Fromm [AMR **3**, Rev. 292]. Experimental method is described in detail for finding liquid rigidity by Reiner [AMR **4**, Rev. 1159], or shear modulus G of Maxwell liquids with the rheological equation $\gamma = \dot{\tau}/G + \tau/\eta$. Liquids tested are dibutylphthalate, dimethylphthalate, diethylphthalate, glycerin and water (39%–87%), oil, and tar. In the first approximation, all show the property of elastico-viscosity; refined analysis, however, suggests the introduction of the concept of "anti-Maxwellian" liquids, with negative shear modulus $G < 0$. This would generalize the picture: $G > 0$ (Maxwellian), $G = \infty$ (Newtonian), $G < 0$ (anti-Maxwellian fluid). Temperature range is 20 C to 60 C.

Author promises further experimental results, which, according to this reviewer, are necessary before final conclusions can be made.

V. G. Szebehely, USA

3097. Weill, A. R., Mme., On the mechanism of plastic deformation of metals as a function of temperature and velocity, after recent works by W. A. Wood (in French), *Mém. Artill. fr.* **26**, 1, 41–57, 1952.

Metallurgical and x-ray observations on polycrystalline aluminum by W. A. Wood [AMR **3**, Revs. 477, 480; AMR **4**, Revs. 206, 2918] are reviewed in detail. Wood's proposed mechanism of deformation by formation of crystallites is discussed. Author shows that data can be interpreted simply in terms of formation, movement, and orientation of dislocation, thus resolving the objections of critics who feel that the work is not in agreement with recent theories of dislocations and polygonization. Author comments that Wood's mechanism is workable and worthy of consideration. Author cautions investigators not to ignore such theories simply because they are not explained in terms of the "magic" dislocation theory.

A. D. Schwope, USA

3098. Benthem, J. P., On the stress-strain relations of plastic deformation, *Nat. LuchtLab. Amsterdam Rap.* S.398, 88 pp., 13 figs., Dec. 1951.

Following a section introducing tensor theory, this survey takes up stress and strain tensors and the use of stress invariants. The stress-strain relations of both finite and incremental type for both hardening and nonhardening materials are then developed and discussed critically, with major emphasis on isotropic strain-hardening. The anisotropic hardening theory of Batdorf and Budiansky is introduced in an original manner in terms of a two-dimensional analog which reviewer feels is lacking in physical significance, but which nevertheless exhibits rather simply many of the principal features of the theory.

Paper is considerably shorter than the similar survey by Drucker [AMR **4**, Rev. 3553], both because of its more succinct style and because it is less complete. Among the topics omitted are the work of the Russian plasticians and the laws for initially anisotropic materials. In common with a number of other writers on plasticity, author, in reviewer's opinion, is not sufficiently cognizant of the distinction between theories capable of making predictions and therefore subject to experimental test on the one hand, and, on the other, general mathematical formulations readily adjustable to anything that happens (and equally readily adjustable to anything that doesn't) and therefore not subject to experimental test. In the latter class are the more general theories of both finite and incremental type when applied to experiments in which the principal axes of stress remain fixed and stress components increase proportionally. Despite these limitations, however, paper provides a generally excellent summary and critical evaluation of theories and recent experiments on plastic stress-strain relations.

S. B. Batdorf, USA

3099. Stüssi, F., The deformation of fossils (in French), *Schweiz. Bauztg.* **68**, 38, 523–526, Sept. 1950.

3100. Cottrell, A. H., Formation of immobile dislocations during slip, *Phil. Mag.* (7) **43**, 341, 645–647, June 1952.

Lomer's work [*Phil. Mag.* **42**, 1327, 1951] on the formation of new dislocations from certain dislocations on intersecting slip planes in the face-centered cubic lattice is extended. Author shows that the new dislocations may give rise to groups of imperfect dislocations which may be unable to move due to the arrangement of their associated stacking faults. The possible importance of such immobile dislocations in work-hardening is pointed out.

H. H. Hilton, USA

Failure, Mechanics of Solid State

(See also Revs. 3036, 3120)

3101. Teed, P. L., Fatigue of aircraft materials with special reference to micro-structure, *J. roy. aero. Soc.* **56**, 498, 427–437, June 1952.

Author illustrates the effect of metallographic structure with various microphotos and graphs, suggesting that fatigue failure is caused by localized transitory plastic flow, giving rise to work-hardening and, finally, cracking. Plastic flow at stresses below the engineer's elastic limit would be due to local (and often extremely local) stress concentrations that can arise from random arrangement of elastically anisotropic crystals, intercrystalline boundaries (which are stronger but less ductile than the crystals), mixed phases of different elastic characteristics, precipitation hardening (especially as the precipitate does not occupy the same volume as it did when in solution), cold work, residual stresses, discontinuities (microcracks and cavities), and inclusions (which can be malign or benign, depending on physical properties and shape).

Examples are given of some precipitation-hardening aircraft alloys, showing that the tremendous rise in yield stress due to precipitation hardening is accompanied by only a very modest rise in endurance limits. It is further shown that the fatigue

strength of fine-grained material (other than the exception, ferritic steel) is better than that of coarse-grained material. Author does not see structural metals as a largely ordered array of atoms, making possible a treatment on mathematical-physical grounds. In his view, supported by the examples of his paper, the complexity of stress distribution within any metallic compound is such that, to avoid all risk of its failure by fatigue, realistic testing is the sole insurance.

J. H. van der Veen, Holland

3102. **Ros, M., Fatigue of metals** (in German), *ZAMM* **32**, 4/5, 130–145, Apr./May 1952.

Paper reports results of careful study of fatigue of metals, with notched and unnotched samples, and under homogeneous and nonhomogeneous, uniaxial and nonuniaxial stress. Alternating stress produces anelastic and plastic disarrangements of lattice, which increase with number of stress reversals and value of stress amplitude. These disarrangements locally decrease lattice cohesion, loosen structure, and finally produce fissures. Thus, the various phases of rupture by fatigue are: (a) Formation of first fissure; (b) growth of fissure without appreciable deformation; (c) forced rupture of reduced and weakened cross section. Exhaustion by fatigue does not differ fundamentally from repeated static stressing; however, each new stressing finds the conditions of the sample altered in consequence of local structural changes. Resistance to fatigue is not affected by time rate of stressing unless the behavior of a sample is influenced by work-hardened or otherwise treated surface layer. The Coulomb-Mohr hypothesis has been found generally valid. B. Gross, Brazil

3103. **Weibull, W., Statistical design of fatigue experiments,** *J. appl. Mech.* **19**, 1, 109–113, Mar. 1952.

Author's method is based on the proposed equation: $S - E = A(N + B)^{-m}$, S being the applied stress, N the fatigue life, and E the endurance limit. To determine E, A, B and m, four different testing stresses are necessary. After discussing a graphical method, a method for computing the parameter values is given. Author develops a series of analytical expressions for the partial derivatives of the parameters with respect to the quantities *stress* and *fatigue life*. Using these expressions, he calculates the variance of E caused by the scatter of loads and fatigue lives, and deduces a set of formulas for the optimum distribution of a given number of tests over the four stress levels. The optimum choice of stress levels is also discussed, and it is shown that the variance of E is strongly dependent on the choice of the lowest stress. A minimum variance is obtained when this stress is slightly above the upper endurance limit. At this level, about 40% of the scatter in fatigue life is shown to be caused by the uncertainty of load readings. At high stress levels, this factor is of minor importance. The application of the formulas is illustrated by an actual test series on aluminum alloy 75S. Testing time and costs may be reduced by more than 40% by using the formulas.

J. H. van der Veen, Holland

Material Test Techniques

(See also Revs. 3079, 3108)

3104. **Markl, A. R. C., Fatigue tests of piping components,** *Trans. ASME* **74**, 3, 287–299, Apr. 1952.

Fatigue tests have been performed on over 400 piping components including short and long radius turns, elbows, miter bends, tees, and straight sections of tubing. Most components were of 4-in. standard weight and include a variety of types of fabrications. The objective was to present practical data and empirical equations giving the effective stress-intensification factors for the above-shaped components relative to the straight section. The experimental results are presented as $S - N$ plots, in which the stress S is a nominal value determined from the ordinary beam formula and the applied bending moment, and N represents the number of cycles of stress (full stress reversal). A straight section of pipe was assumed to have a stress-intensification factor of unity. For piping of similar sizes but of other shapes, the stress-intensification factor was determined as the ratio of the nominal stresses required to fail the straight section and the component under study after a given number of cycles of stress. This ratio was reasonably constant over a wide range of cycles to failure. (Reviewer notes that this stress-intensification factor may be different from that determined by other methods of stress analysis.) At some sacrifice in accuracy, a simple equation is presented which gives the stress-intensification factor for many of the above shapes. This paper completes a series of three on this project [AMR **1**, Rev. 75 or 1220; **3**, Rev. 2626].

Irwin Vigness, USA

3105. **Gillemot, L., Measuring devices for investigation of true stresses** (in German), *Acta Techn. Hung., Budapest* **1**, 3, 191–197, 1951.

Author describes ingenious machine for plotting true stress directly while testing rubber and rubberlike materials. Determination is made on basis of change in length of test specimen and assumption that volume remains constant. Accuracy of results depends upon uniformity of elongation throughout gage length. Frank J. Mehringer, USA

3106. **Schinn, R., and Wolff, Ursula, Some results of ultrasonic testing of forgings by the method of impulse echo** (in German), *Stahl u. Eisen* **72**, 12, 695–702, June 1952.

Figures appearing on the oscillograph screen when testing big forgings with the reflection method are shown. These "reflectograms" are compared with macroscopic photographs and magnetic powder pictures of the defect. Applied to the cast block, it does not give satisfactory results; however, after the first forging step of the cast block, the ultrasonic inspection method can be applied with good success, and can be used after each subsequent step of manufacturing.

Paper will be of interest to personnel working directly with the ultrasonic flaw detector. R. O. Fehr, USA

Mechanical Properties of Specific Materials

(See also Revs. 3071, 3091, 3093, 3101, 3102, 3105, 3259)

3107. **Bastien, P., and Winter, C., Mlle., Influence of heat-treatment on the true tensile curves for mild steel** (**French research results**), *Metal Treatm.* **19**, 80, 213–216, May 1952.

Condensation of a paper read before the Société Française de Métallurgie, in which are presented results of true stress-strain curves of three mild, open-hearth steels, in normalized, overheated, or quenched and aged conditions, interpreted in terms of parabolic relation between stress and strain, with reference to capacity for deep drawings. G. V. Smith, USA

3108. **Hughes, D. E. R., The hot-torsion test for assessing hot-working properties of steels,** *J. Iron Steel Inst.* **170**, part 3, 214–220, Mar. 1952.

Hot torsion tests were made on mild-steel and on high-carbon chromium-steel specimens with $^3/_8$-in. diam and $1^1/_2$-in. length at temperatures from 950 to 1400 C with 12 to 600 rpm, respectively, and a speed of deformation from about 10 to 400%/s. In all

cases, a well-marked maximum was found in the curves connecting revolutions to failure with temperature. Except at very low testing speeds (10–50 rpm) for the mild-steel specimens, the temperature at the maximum is that found to be best for the rotary piercing operation in the manufacture of seamless steel tubes. The relationships between torque and temperature show no unexpected features. The curves are displaced to higher torque levels by increasing speed.

Examination of mild-steel and high-sulphur mild-steel specimens that had been deformed only part of the way to fracture showed that the outer regions underwent very heavy deformation, and that a flow effect at right angles to that of the original rod developed ultimately. Specimens cut transversely from a mild-steel billet gave very much lower values for revolutions to failure than longitudinal ones from the same billet.

E. Siebel, Germany

3109. Steele, M. C., and Young, J., An experimental investigation of over-straining in mild-steel thick-walled cylinders by internal fluid pressure, *Trans. ASME* **74,** 3, 355–361, Apr. 1952.

Authors compare results of experiments on open-ended tubes with predictions of the total strain theories. Material used is hot-rolled billet, machined and annealed at 1630 F. Bore circumferential strains were measured at four or six symmetrical positions in a transverse section, and axial and circumferential strains at similar locations on the outer surface. Lüders' lines on the ends were examined at various stages of loading. Mechanism of yielding is formation of wedge-shaped regions of plastically deforming material between the bore and outer surface. Wedges are located asymmetrically and bounded by maximum shear stress trajectories. Tubes do not deform uniformly, and, as would be expected, predictions of theory for isotropic uniformly expanding tubes overestimate the fully plastic pressure. Continued deformation at constant load occurs over long periods, as exemplified by increase of bore circumferential strain of 20% in 30 minutes. The results are interesting, especially in view of previous work. Reviewer would like to see a more comprehensive series of tests with checks on anisotropy, axiality of bore, etc. Paper recalls the need for extending the current mathematical theory to cover questions of stability and uniqueness.

J. F. W. Bishop, England

3110. Buttner, F. H., Funk, E. R., and Udin, H., Viscous creep of gold wires near the melting point, *J. Metals,* **4,** 4, 401–407, Apr. 1952.

Gold wires, 5 mil in diameter, are found to creep viscously up to approximately 4×10^5 dynes per sq cm around 1300 K. The average viscosity coefficient is found to be 18.9×10^{12} poises. This experimental value is compared with calculations based on Herring's equation [*J. appl. Phys.* **21,** p. 437, 1950] which give a viscosity coefficient of 13.6×10^{12} poises.

The creep behavior can be divided into two ranges, a microcreep and a macrocreep range, which are separated by a macroyield point according to the convention of Chalmers for the creep of tin. The wires creep by means of two additive mechanisms in the macrocreep range after the fashion qualitatively presented by Kauzman [*Trans. AIME* **143,** p. 37, 1941]. It is also shown that when both mechanisms occur simultaneously they affect one another.

From authors' summary by Aris Phillips, USA

3111. Holt, Marshall, and Hartmann, E. C., Fatigue tests on aluminum alloy spot-welded joints, *Welding J.* **31,** 4, 183s–187s, Apr. 1952.

Authors present results of tension-compression fatigue tests of spot-welded lap joints of aluminum sheets 0.064 in. thick. Nine different types of specimens were tested, each type of a different alloy or temper. The points on the S-N curves cover the range from 1 cycle to 10^8 cycles to failure. These tests show that: (a) The various alloys and tempers have approximately the same fatigue strength. (b) Aging of specimens after spot-welding does not improve the fatigue strength. (c) There is no correlation between the static and fatigue strength. (d) Single spot welds are inferior to single rivets as regards fatigue strength.

Alexander Yorgiadis, USA

3112. Marin, J., and Hughes, W. P., Fatigue strengths of 14S-T4 aluminum alloy subjected to biaxial tensile stresses, *NACA TN* 2704, 24 pp., June 1952.

Tests were made on thin-walled tubular specimens of 14S-T4 under a combination of fluctuating axial tension and fluctuating internal pressure. Ratios of circumferential stress to axial stress were 0, $^1/_2$, 1, and 2, S-N curves being given up to about 5×10^6 cycles. Results show pronounced influence of anisotropy of material so that it is impossible to verify failure theories. Testing equipment and method are fully described.

F. J. Plantema, Holland

3113. Majors, H., Jr., Dynamic properties of nodular cast iron—Part I, *Trans. ASME* **74,** 3, 365–380, Apr. 1952.

Paper presents results of fatigue and damping capacity tests on magnesium-treated cast iron in the annealed and as-cast condition. The fatigue tests (rotating cantilever) were run at 200 and at 6000 rpm. The S-N curves for 200 rpm were below those for 6000 rpm in the higher stress range except for the notched as-cast condition. For both speeds, the S-N curves leveled off at the same value. Three sizes of notched and plain bars were used to check the effect of the depth of notch on the endurance limit. The tests indicate that a maximum in the dynamic stress-concentration factor occurs when the depth of the notch is about $^1/_4$ of the shank diameter. Evidence for this is obscured somewhat by poor coding of the test points. Specific damping capacities obtained in static direct tension-compression, in static bending, and in dynamic torsional tests are compared. Damping capacity varied with the method of testing. The specific damping capacity of magnesium-treated cast iron is compared with that of an SAE 1020 steel and with cast iron melted in a cupola. Some interesting discussions on the change of damping energy with stress level and number of cycles and on the general properties of nodular cast iron are published with the paper.

Evan A. Davis, USA

3114. Pearson, R. G., The sampling of timber for standard mechanical tests, *Austral. J. appl. Sci.* **3,** 1, 25–52, Mar. 1952.

Data are presented as to how an efficient sample size may be determined for mechanically testing wood satisfactorily. Full random sampling is considered essential. Selection of one specimen per tree is considered sufficient and most economical.

Reviewer believes that complete randomization, as considered essential by author to eliminate bias, may lead to the inclusion of nonrepresentative material, such as compression wood, and, thus, to misrepresentation. Furthermore, such randomization may not always provide maximum possible correlation within the data.

E. G. Stern, USA

3115. Curry, W. T., The effect of moisture content on the tensile strength of Sitka spruce, *Aircr. Engng.* **24,** 279, 142–143, May 1952.

From 1600 separate tests, the effect of moisture on the tensile strength is seen to vary with density. A method is suggested whereby the tensile strength can be determined with reasonable accuracy.

From author's summary

3116. Lincoln, B., Flexural fatigue and visco-elastic properties of wool fibres, *J. Text. Inst. Trans.* **43,** 4, T158-T172, Apr. 1952.

Author discusses relation between the resistance of fibers to repeated bending and their viscoelastic properties. Experiments are described on measurements of flexural fatigue and damping capacity on wool fibers before and after a carbonizing treatment. It is found that decrease in resistance to flexural fatigue is associated with increased internal damping.

H. Kolsky, England

3117. Hemelrijk, J., Some tests for hypotheses concerning tensile strength of cloth, *Appl. sci. Res.* (*A*) **3,** 3, 211–224, 1952.

Author presents a method of applying a system of statistical testing hypotheses to a physical problem in which there are two related variables. He assumes that the independent variable is subject to little or no error, while the dependent variable is subject to random errors. He applies these criteria to the case of textile fabric strength for illustrating application of method. Based on certain physical assumptions, author sets up three testing hypotheses. These are tested progressively. Rejection of any hypothesis rejects the theoretical assumptions. Acceptance of all three hypotheses is necessary to accept the assumptions.

Reviewer believes the method is useful in engineering problems, but that author has not selected a good illustration for application. Acceptance or rejection of hypotheses based on assumptions is dependent on the soundness of the original assumptions. Author draws on a textile test theory dated 1926, one which has subsequently been considerably modified. Little account is taken of the effect of the type of test instrument in the assumptions. A pendulum-type tester with a constant-velocity traversing jaw whose velocity can be changed through a gear boss is employed. Experimental criterion is rupture load at three different times to rupture. Due to the nature of the pendulum tester, the result is three different testing rates which are not proportional to the change in times to rupture. This invalidates the basic theory.

Author's application is clear and can be used for other types of experiments, provided present paper is viewed only as an illustration of method of application of testing hypotheses.

Rogers B. Finch, USA

3118. Low, J. R., Jr., The effect of quench-aging on the notch sensitivity of steel, *Welding J.* **31,** 5, 253s-256s, May 1952.

Charpy impact-test specimens of a semi-killed 1020 steel were quenched from 690 C and aged at room temperature for periods up to three years. During this period the transition temperature increased from −40 C to 0 C. Over-aging at 350 C, either immediately following quenching or after three years' aging, lowers the transition temperature to −15 C. It is suggested that quench-aging is responsible for the commonly observed brittle zone adjacent to welds in this grade of steel, and that a low-temperature postheat treatment similar to that used for over-aging should improve the low temperature ductility of welded structures made of this and similar grades of steel. It is also shown that a decrease in the cooling rate from 690 C raises the transition temperature; this effect is believed to account for the fact that as-rolled plates show an increase in transition temperature as the thickness increases.

From author's summary

3119. Lee, E. H., Plastic flow in a V-notched bar pulled in tension, *J. appl. Mech.* **19,** 3, 331–336, Sept. 1952.

Paper presents analysis of flow in an ideally plastic, deeply notched bar, pulled in tension in plane strain. It sets up equations of flow and derives stress field and deformation field as function of notch angle, notch dimensions, and shear stress.

Experimental check is obtained by scribing a square grid on an undeformed bar and observing pattern at regular time increments. Author points out that plastic flow occurs prior to fracture only if the maximum tensile stress produced by notch constraint, which is derived, is less than the technical cohesive strength. Under such conditions, the greatest stress resulting from notch is a function of the yield stress.

Paper is useful in evaluating notch-bar tension tests and in understanding previous theories of Kuntze on the subject.

H. I. Fusfeld, USA

3120. Libsch, J. F., Powers, A. E., and Bhat, G., Temper embrittlement in plain carbon steels, *Trans. Amer. Soc. Metals* **44,** 1058–1068, 1952.

Specimens from AISI 1050 steel (0.46% C, 0.75% Mn) hardened by heating to 830 C and brine-quenching were tempered in the range of 455 to 680 C for times varying from 5 sec to 100 hr. By means of Charpy impact tests, transition temperature was determined for differently tempered specimens. This temperature increases with tempering temperature and time. From results obtained for various combinations of embrittling temperature and time, iso-embrittlement lines are derived which author regards as the fundamental basis for evaluating the influence of alloying elements upon temper embrittlement.

Heinrich Mussmann, Germany

3121. Neighbours, J. R., Bratten, F. W., and Smith, C. S. The elastic constants of nickel, *J. appl. Phys.* **23,** 4, 389–393, Apr. 1952.

The longitudinal and transverse acoustic wave velocities were determined by the pulsed ultrasonic method. Wave velocities were measured in a transverse magnetic field so that the results are characteristic of magnetically saturated nickel. Velocity measurements were corrected for transit time error. The orientation of the crystallographic axes was measured by the back-reflection x-ray method. The elastic constants were then calculated by a perturbation method. The values of the constants reported are: $C_{11} = 2.53$, $C_{12} = 1.52$, and $C_{44} = 1.24$ in units of 10^{12} dynes per sq cm.

Evan A. Davis, USA

3122. Baker, W. O., and Heiss, J. H., Interaction of polymers and mechanical waves, *Bell Syst. Tech. J.* **31,** 2, 306–356, Mar. 1952.

The mechanical properties of polymer materials depend on the amount of motion and the relaxation time of various segments of the polymer chains. By using shear and longitudinal ultrasonic waves over wide enough frequency ranges to include the thermal vibrations of significant groups or segments of the macromolecules, the relaxation spectra of liquid and solid polymer materials can be investigated.

It is shown that these relaxation spectra are closely connected with the chemical structure of the polymer materials. Natural rubber and silicone rubber show a small change of stiffness with frequency and a relatively low dissipation. More complicated chains with greater sterical hindrance show a large stiffness that increases rapidly with frequency, and a high dissipation. Some of these materials, such as polyisobutylene, serve as excellent dampers.

When long chain molecules are placed in solution, characteristic relaxations are found which are associated with various segment motions that single chains can undergo. In polyisobutylene, three relaxations have been isolated which are associated with the configuration motion of the molecule as a whole, with a transient entanglement process, and with the smallest chain segment

motion. These relaxations differ depending on the solvent and on the chain structure. The two upper ones can be traced in polymer liquids and in linear polymer solids.

Warren P. Mason, USA

3123. Cox, H. L., The elasticity and strength of paper and other fibrous materials, *Brit. J. appl. Phys.* **3,** 3, 72–79, Mar. 1952.

An analysis is made of the effect of orientation of the fibers on the stiffness and strength of paper and other fibrous materials. It is shown that these effects may be represented completely by the first few coefficients of the distribution function for the fibers in respect of orientation, the first three Fourier coefficients for a planar matrix, and the first fifteen spherical harmonics for a solid medium. For the planar case, it is shown that all possible types of elastic behavior may be represented by composition of four sets of parallel fibers in appropriate ratios. The means of transferring load from fiber to fiber are considered, and it is concluded that the effect of short fibers may be represented merely by using a reduced value for their modulus of elasticity. Results of the analysis are applied to certain samples of resin-bonded fibrous-filled materials, and moderately good agreement with experimental results is found.

From author's summary by H. Kolsky, England

Mechanics of Forming and Cutting

(See also Revs. 3033, 3054, 3106)

3124. Geleji, A., Calculation of the forces acting in the roll gap (in German), *Acta techn. Hung., Budapest* **2,** 1, 123–142, 1951.

Article deals with a method of computing the metal resistance force between the rolls and the torque required to turn the rolls during rolling of metals.

Some simplifying assumptions have been made; e.g., the resistance to deformation is constant between the rolls, and the coefficient of friction is known. Then, by means of a combination of analytical and graphical procedures, one starting from the entry side of the rolls and the other from the exit side, author proceeds to plot curves which intersect at a point of no slip. Once this point is determined, however, his analysis is quite straightforward, and the results seem to have practical value. He has applied his method to the rolling of high purity aluminum, duralumin, nickel steel, and ordinary carbon steel with results, he concludes, that are consistent with actual behaviors.

R. G. Sturm, USA

Hydraulics; Cavitation; Transport

(See also Revs. 3095, 3096, 3217)

3125. Escande, L., Sustained oscillations in surge tanks with throttling (in French), *Rev. gén. Hyd.* **17,** 64, 188–195, July/Aug. 1951.

An abbreviated paper by the author was published previously [see AMR **5,** Revs. 1765, 2400]. This extended paper contains the detailed analytical treatment and the graphical calculations, the general conclusions of which were presented previously.

Charles Jaeger, England

3126. Milham, R., and Catheron, A. R., Valve control of liquid flow, *Instruments* **25,** 5, 596–598, 634–640, May 1952.

Selection of a valve type and size for a flow-control problem involves analysis of valve characteristics. The drop ratio in a system changes the characteristics of a valve and affects control quality. These significant factors are presented in practical form.

From authors' summary

3127. Crausse, É., On the study of bottom-gates with reduced magnitudes (in French), *C. R. Acad. Sci. Paris* **234,** 11, 1126–1128, Mar. 1952.

Energy equation $E_1 = q^2/2ga^2 + na$, and momentum equation $F_1 = q^2/ga + n^2a^2/2$ (F_1 is specific energy, q rate of flow per unit width of canal, a thickness of "vena contracta," na depth on vena contracta section, and F_1 momentum per unit width) are used. Author puts these equations in dimensionless form, referring energy to critical depth and momentum to square of critical depth.

Two charts are presented: (1) A double diagram of these dimensionless values versus depth. Author uses Prof. Escande's hypothesis: There are no losses of head between gate and vena contracta; all head losses are comprised between vena contracta and end of jump; parallel flow within vena contracta. The diagram presents several curves with n as parameter. (2) The second diagram is derived graphically from the first, and gives the relation between head upstream gate and tailwater level for several values of n and a.

Author announces future publications on this subject.

A. Balloffet, Argentina

3128. Crausse, É., On a property of a horizontal liquid flow in a uniform channel (in French), *C. R. Acad. Sci. Paris* **234,** 22, 2152–2153, May 1952.

Author again takes specific energy and momentum equations for rectangular canal (see preceding review). When these relationships are divided by critical depth h_c and h_c^2, respectively, he obtains $E' = E/h_c = 1/2a'^2 + na'$; and $F' = F/h_c^2 = 1/a' + n^2a'^2/2$. Here a' is flow depth divided by h_c.

It is observed that E' is transformed in F', and reciprocally, when a' is replaced by $1/na'$. Then, each flow with depth equal to a' corresponds to a conjugated one with $1/na'$. Specific reduced energy of one of these flows equals reduced momentum of the other, and reciprocally. Graphical interpretation is given.

A. Balloffet, Argentina

Incompressible Flow: Laminar; Viscous

(See also Revs. 3035, 3210, 3232)

3129. Arzhaníkh, I. S., Functions of the stress tensor in hydrodynamics (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **83,** 2, 195–198, Mar. 1952.

Stress functions in statics of an elastic body, which were introduced by Maxwell and Morera, have been applied from various points of view by Kuz'min [*Doklady Akad. Nauk SSSR* **49,** p. 335, 1945], Blokh [AMR **4,** Rev. 1039], and Filonenko-Borodic [AMR **4,** Rev. 4094; *Prikl. Mat. Mekh.* **15,** 5, 563–574, Sept.-Oct. 1951]. The great advantage of the Maxwell-Morera formulas is that they satisfy the equilibrium equations identically.

In the paper under review, analogous formulas satisfying identically the equations of hydrodynamics are derived. The formulas obtained, which in the equilibrium case reduce to the Maxwell-Morera formulas, are too lengthy to be reproduced here.

Eugene Leimanis, Canada

3130. Birkhoff, G., A new theory of vortex streets, *Proc. nat. Acad. Sci. Wash.* **38,** 5, 409–410, May 1952.

For any plane flow satisfying the Navier-Stokes equations, the mean transverse moment of the vorticity per unit length is a con-

stant in time. Starting with this result, a new theory of the wake behind an obstacle is briefly outlined for a nonviscous fluid.

A. R. Mitchell, Scotland

3131. Neumark, S., Pressure distribution on an airfoil in nonuniform motion, *J. aero. Sci.* **19**, 3, 214–215, Mar. 1952.

For pressure distribution along a thin two-dimensional airfoil in an unsteady vertical motion, author derives formula similar in form to well-known formulas for lift and moment, given first by von Kármán and Sears [title source, **5**, 10, 378–390, Aug. 1938]. These formulas, relating quantities mentioned to the vorticity (on the airfoil and in the wake), each consist of three terms, viz., a quasi-steady part, an apparent mass contribution, and a contribution due to the wake. Application is made to case of steady oscillations and, in particular, to vertical translatory oscillation.

A. I. van de Vooren, Holland

3132. Gerber, R., On the existence of irrotational, plane periodic flows of an incompressible heavy liquid (in French), *C. R. Acad. Sci. Paris* **233**, 21, 1261–1263, Nov. 1951.

Author announces that, under certain restrictions, he is able to prove the existence of two-dimensional periodic gravity waves over a symmetric periodic bottom. The restrictions, aside from those mentioned in the title, are a limitation on the maximum slope of the bottom and on the value of a certain Froude number. The associated conformal-mapping problem is reformulated as an integrodifferential equation which can be interpreted as an equation $x = F(x)$ in a Banach space, where F is completely continuous. The restrictions mentioned allow application of the methods of Schauder and Leray. No details are given.

J. V. Wehausen, USA

3133. Rumer, Yu. B., The problem of a submerged jet (in Russian), *Prikl. Mat. Mekh.* **16**, 2, 255–256, Mar./Apr. 1952.

Author considers solutions of the Navier-Stokes equations in spherical coordinates which are regular in θ and of the following form:

$$v_r = r^{-1}F_1(\theta) + r^{-2}F_2(\theta),\ v_\theta = r^{-1}f_1(\theta) + r^{-2}f_2(\theta)$$
$$v_\varphi = 0,\quad p/\rho = r^{-1}g_1(\theta) + r^{-2}g_2(\theta)$$

The functions F_1, f_1, and g_1 were determined earlier by Yatsev [AMR **4**, Rev. 3597]. The author determines F_2, f_2, and g_2 corresponding to a particular choice of the F_1, f_1, and g_1, namely that resulting when $a = b = c = 0$ (see the cited review for notation). This may be interpreted as the flow resulting when a pipe discharges into a space filled with the same fluid.

J. V. Wehausen, USA

3134. Birkhoff, G., Induced mass with free boundaries, *Quart. appl. Math.* **10**, 1, 81–86, Apr. 1952.

Using the properties of the acceleration potential A, author examines the concept of induced (added) mass in the case of a liquid with a free surface, assuming only incompressibility and neglecting gravity. The following results are proved for the initial instant of motion: (1) Consider a region R filled with liquid, bounded in part by a wetted surface W and in part by a free surface S at constant pressure. Defining "acceleration kinetic energy" as $T = \frac{1}{2}\rho\int\int\int_R \nabla A\cdot\nabla A dR$ (ρ is mass density), it is shown that T is minimized by the free-surface condition $A = 0$ on S relative to all other volume-conserving flows in R satisfying the condition $a_{\text{normal}} = \partial A/\partial n$ on W (a is acceleration). (2) In a fluid motion with undisturbed velocity $U = U(\bar{x}, t)$ altered at time $t = 0$ by an additional normal acceleration $f(x)$ of W, the instantaneous pressure distribution required to accelerate the fluid is the same as if the fluid were at rest. Furthermore, the impulsive pressure required to produce an additional normal "impulsive velocity" of W is $-\rho A$.

Consider next a liquid region bounded by a wall W'' (container) and the wetted surface of an accelerated missile W' in the free surface S. Using the definition of induced mass tensor, $T_{hk} = T_{kh} = \rho\int\int\int_R \nabla A^h\cdot\nabla A^k dR$, where the A^h and A^k are acceleration potentials for unit translations (accelerations) of W' parallel to the axes and unit rigid rotations of W' about the coordinate axes, the following are proved: (1) The diagonal components T_{hh} are increased if either the region occupied by liquid is increased or free-surface area is replaced by container walls or wetted missile area. (2) Let a volume ΔV of liquid be replaced by missile. Then the new translation-induced mass tensor T'_{hh} satisfies $T'_{hh} \geqslant T_{hh} - \rho\Delta V$. (3) The tensor component T_{hk} represents the total h-component of initial pressure force required to produce a unit k-acceleration of the missile.

Momentum and moment of momentum interpretations of T_{hk} are also given. Application of the results to the case of impact on water is indicated, and a rough estimate is derived for the effect of gravity on the pressure exerted by a liquid on a missile with an attached cavity.

Phillip Eisenberg, USA

3135. Kalinin, N. K., Filtration through a double-layered wedge (in Russian), *Prikl. Mat. Mekh.* **16**, 2, 213–222, Mar./Apr. 1952.

A dam of wedge-shaped cross section consists of two types of media with different permeabilities. These media are separated by a plane which contains the ridge of the dam. It is required to find the directions and speeds of the flow of the water (which is filtrating through the dam) in each layer of the dam. The problem consists in the determination of two functions of a complex variable, which satisfy certain boundary conditions and each of which is regular within one of two angular regions with a common boundary. The solution is sought and found in the form of Fourier integrals.

Courtesy of Mathematical Reviews H. P. Thielman, USA

3136. Lighthill, M. J., On the squirming motion of nearly spherical deformable bodies through liquids at very small Reynolds numbers, *Comm. pure appl. Math.* **5**, 2, 109–118, May 1952.

The problem is to determine to what extent a deformable spherical body can progress through liquid by performing small oscillations of shape if all Reynolds numbers are so small that inertial forces are negligible compared with forces due to pressure and viscous stress. Discussing motions with axial symmetry and finite total energy, it is found that, to the second order of approximation, the velocity of the center of the body is not an exact time derivative of an oscillatory function, so that with suitable combinations of the possible modes of deformation, gradual progress can be made. Three representative examples of the more efficient kinds of squirming motion are discussed in detail with diagrams. [Another problem, that of swimming by sending lateral waves down a thin tail, is treated by G. I. Taylor; AMR **5**, Rev. 1905.]

L. M. Milne-Thomson, England

3137. Nevzglyadov, V. G., On boundary conditions of the new method in the dynamics of a viscous fluid (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **82**, 2, 213–216, Jan. 1952.

Author reports continuation of previous work on flow of a viscous fluid past a plate. While outer flow (a distance $\pm\lambda$ above and below plate) satisfies the Oseen equation, inner flow must satisfy the Navier Stokes equations, vanish on the plate, and the velocity and pressure must be continuous across the interface. Starting from a linear velocity profile in the boundary layer de-

fined here, author finds the shape of the boundary layer and wake and a corrected drag of $1.3/R_x^{1/2}$. The results appear to depend critically on the approximation chosen for the profile.

Leon Trilling, USA

3138. Sakadi, Z., Motion of an incompressible viscous fluid between two concentric spheres, *Math. Japonicae* **2,** 71–74, 1951.

In the title problem, one sphere is fixed and the other rotates with constant angular velocity. Author carries through the solution, retaining terms of the second order in the velocities. He finds that the value of the moment on the fixed sphere is the same as in the linearized theory [cf. Lamb, "Hydrodynamics," 6th ed., Cambridge, 1932, §334]. Retention of third-order terms would have shown a change [cf. Toraldo di Francia, AMR **5,** Rev. 169].

J. V. Wehausen, USA

3139. Ballabh, R., On a fluid motion with a spherical boundary, *Proc. nat. Inst. Sci. India* **17,** 123–126, 1951.

In an earlier work [*Proc. Benares math. Soc.* **2,** 69–79, 1940], author has stated conditions for two flows of an incompressible viscous fluid to be superposable in the sense that the vector sum of the two velocities be again the velocity vector of a flow. He applies these conditions to study flows superposable on a uniform flow and having a spherical stream surface as boundary. He derives in this way an exact solution of the Navier-Stokes equations valid within and without a spherical stream surface, except for a singularity at the center. The flow in the exterior is not uniform at infinity.

Courtesy of Mathematical Reviews D. Gilbarg, USA

3140. Heisenberg, W., On the stability of laminar flow, Inter. Congr. Math., Cambridge, 1950, Conf. appl. Math., 292–296.

Author reviews briefly and critically the history of problem of stability in two-dimensional flow. He interprets mathematical results physically in terms of statistical theory of isotropic turbulence.

Irvin M. Krieger, USA

3141. Yeh, H., Secondary flow in cascades, *J. aero. Sci.* **19,** 4, 279–280, Apr. 1952.

Author obtains, in a simple and direct manner, a result previously obtained by Squire and Winter [AMR **4,** Rev. 3672] for a first-order approximation of the secondary flow that is obtained downstream of a cascade of airfoils when the entering flow is parallel but not uniform in the spanwise direction.

Peter Chiarulli, USA

Compressible Flow, Gas Dynamics

(See also Revs. 3190, 3192, 3204, 3211, 3227)

3142. Maslen, S. H., Supersonic conical flow, *NACA TN* 2651, 32 pp., Mar. 1952.

Equations of motion for a perfect isoenergetic fluid are expressed in spherical polar coordinates; for conical flows with vertexes at the origin, there is no dependence on radial distance. Resulting equations are of mixed type and are solved by method of characteristics in hyperbolic regions. In elliptic regions, a modified relaxation method is employed in which only second derivatives are expressed in finite difference form, the finite difference equations having variable coefficients involving first derivatives. Numerical solution is generally straightforward, the main difficulty arising from the initially unknown positions of shock waves: Shock waves separating two hyperbolic regions are found by standard methods; shock waves separating hyperbolic and elliptic regions are estimated initially by suitable approximate methods and have to be corrected as solution proceeds. Amount of labor is reduced considerably by taking linearized solution to problem as an initial basis for relaxation.

Methods are illustrated by taking as example the case of a flat-plate, zero-thickness, triangular wing with supersonic leading edges. Angle of attack chosen is 12°. Comparison of numerical and linearized solutions is of very great interest: Pressure distributions do not agree well, and extent of hyperbolic and elliptic regions is very different.

G. N. Ward, England

3143. Ludford, G. S. S., The boundary layer nature of shock transition in a real fluid, *Quart. appl. Math.* **10,** 1, 1–16, Apr. 1952.

Family of solutions of Navier-Stokes equations is constructed having large derivatives normal to an arbitrary surface $S(x, y, z, t) = 0$ and small parallel derivatives. The Rankine-Hugoniot shock conditions are satisfied approximately, and the parallel velocity is conserved normal to S. For vanishing viscosity, S becomes a discontinuity surface and the Rankine-Hugoniot conditions become exact.

S. A. Schaaf, USA

3144. Cabannes, H., Determination of the attached shock wave when the velocity downstream of the apex is subsonic (in French), *Actes Coll. inter. Mécan.* **II,** *Publ. sci. tech. Min. Air, Paris,* no. 250, 181–196, 1951.

Author considers plane flow past a sharp-nosed symmetrical obstacle with attached curved shock. It is known, from Crocco's original investigations of this problem, that within a narrow range of Mach numbers, $M_0(\Psi) < M < M^*(\Psi)$ (where 2Ψ is the angle contained by the nose), the flow near the nose behind the shock is subsonic; further, that this flow can be defined analytically in the upper part of the range but is nonanalytic in the lower part of the range. Author extends the investigations of Guderley and Thomas into these two subsidiary ranges. He introduces a third critical Mach number $M_r(\psi)$, where $(r - 1)$ is the order of the lowest nonvanishing derivative of the curvature of the boundary adjacent to the nose. He represents the equations of the shock by a series expansion of x, the coordinate in the undisturbed flow direction, in powers of y, the transverse coordinate. When $M < M_r$, this series is analytic with second term of degree $r + 1$; where $M > M_r$, it is nonanalytic with second term of nonintegral degree $\lambda < r + 1$. In the limiting case when the wedge has straight faces, the nonanalytic representation applies throughout the range, with λ increasing from 1 to infinity as M increases from M_0 to M^*.

Maurice Holt, England

3145. Travers, S., The present state and value of the hydrothermodynamic theory of explosions and shocks. II. Thickness of shock waves and the mechanism of ignition in combustion waves (in French), *Mém. Artill. fr.* **25,** 3, 507–624, 1951.

3146. Travers, S., The present state and value of the hydrothermodynamic theory of explosions and shocks. II. Thickness of shock waves and the mechanism of ignition in combustion waves (in French), *Mém. Artill. fr.* **25,** 4, 923–1010, 1951.

3147. Lawrence, H. R., The lift distribution on low aspect ratio wings at subsonic speeds, *J. aero. Sci.* **18,** 10, 683–695, Oct. 1951.

Author proceeds from E. Reissner's form of the integral equation of lifting surface theory [*NACA TN* 946, 1944], obtains his own variant of this equation, assumes rectangular planform of span $2b$, chord $2c$, and, by making suitable assumptions regarding approximation of $[(x - \xi)^2 + (y - \eta)^2]^{1/2}$ where $|x|$, $|\xi| \leq c$ and

$|y|$, $|\eta| \leq b$, obtains for large Λ(Λ is aspect ratio) the Prandtl lifting-line equation; for very small Λ the result of Jones [*NACA TR* 835, 1946], and by still another approximation, the Weissinger theory [*NACA TM* 1120, 1947] are obtained.

Author then develops two new integral equations for small Λ, these also depending (essentially) on how $[(x - \xi)^2 + (y - \eta)^2]^{1/2}$ is approximated, and on the fact that each is an equation for $k(x) = {}_{b}\int^{b} w(x,y)(b^2 - y^2)^{1/2}\,dy$, whereas in the Prandtl and Weissinger theories an equation for $h(y) = {}_{-c}\int^{c} w(x,y)[(c + x)/(c - x)]^{1/2}dx$ is obtained, where $-w(x,y)$ is airfoil slope. More general planforms are considered briefly.

Calculations using results are made of chordwise pressure distribution, lift-curve slope, aerodynamic center, and damping in roll, of delta and rectangular wings with $\Lambda = 0$ to 4. Results compare well with experiment.

Author speaks of "duality" or analogy between small and large Λ approximations. Reviewer would prefer to speak of these in terms of the symmetry of $[(x - \xi)^2 + (y - \eta)^2]^{1/2}$ in (x,y) and (ξ,η).
William H. Pell, USA

3148. Longhorn, A. L., The unsteady, subsonic motion of a sphere in a compressible inviscid fluid, *Quart. J. Mech. appl. Math.* **5**, part 1, 64–81, Mar. 1952.

A rigid sphere when set in motion causes acoustic energy to be radiated to infinity, in addition to building up the kinetic energy of steady motion. The energy of the compressible fluid varies with the method used to set the sphere in motion. First method is impulsive generation of velocity; second method is gradual acceleration from rest; in both methods, maintenance of steady motion is by a suitable force. Author neglects viscosity, but shows that the additional work for emitting sound waves is done before boundary-layer separation occurs. The work done on the fluid is found to increase due to compressibility, twofold for infinite acceleration and less for finite acceleration. Formulas are based on linearized theory, the order of magnitude of the terms is analyzed, and the effect of higher-order terms indicated.
G. R. Graetzer, USA

3149. Guderley, G., and Yoshihara, H., Two-dimensional unsymmetric flow patterns at Mach number one, *AF tech. Rep.* 6683, 48 pp., Jan. 1952.

Authors extend their work on the symmetrical flow of a sonic stream past a thin double wedge [AMR **4**, Rev. 3299] and consider a wedge set at a small angle of incidence. The simple flow pattern in the symmetrical case is altered by the shift of the stagnation point away from the wedge nose and the creation of a small cavitation region on the upper surface immediately behind it; this complicates the boundary conditions in the hodograph plane. Authors accordingly ignore the cavitation region (they justify this step) and treat the flow at incidence as a small perturbation of the known symmetrical flow. They can then formulate and solve a mixed boundary-value problem of a type studied earlier by Guderley [AMR **5**, Rev. 1149]. The pressure distribution and lift-curve slope are calculated, and the latter is found to have a lower value than that obtained by extrapolation of results of purely supersonic theory.
Maurice Holt, England

3150. O'Keeffe, J., The direct use of Green's method for supersonic potentials, *Quart. J. Mech. appl. Math.* **5**, part 1, 82–92, Mar. 1952.

Paper investigates Green's theorem as applied to a linearized steady supersonic motion past a thin wing. Use of the fundamental solution of supersonic flow in this theorem introduces mathematical difficulties which are not of elementary type. This trouble is circumvented by Volterra by using other particular solutions of the differential equation than the fundamental solution, and by Hadamard by introducing the concept of "finite part" of an improper integral. Author shows that a simple and direct solution of the problem can be constructed by methods strictly analogous to the classical method of Green without using Hadamard's theory.
Tore Gullstrand, Sweden

3151. Goldstein, S., Selected problems in gas dynamics, Inter. Congr. Math., Cambridge, 1950, Conf. appl. Math., 280–291.

Paper contains a very brief survey of linearized approximation for steady supersonic flow and its improvement at singular characteristics and at infinity. Topics discussed are boundary conditions, general theorems, conical flows, sources and sinks, flow in circular tubes, and application of Lighthill's technique [AMR **3**, Rev. 1829] to conical flows and flow at great distances from a body of revolution.
G. N. Ward, England

3152. Klunker, E. B., and Harder, K. C., Comparison of supersonic minimum-drag airfoils determined by linear and nonlinear theory, *NACA TN* 2623, 19 pp., Feb. 1952.

Using linearized theory, Chapman showed [AMR **4**, Rev. 1668] that supersonic airfoils for minimum pressure drag may have blunt trailing edges. Present paper confirms that linearized theory is adequate for purpose of determining optimum shapes (but not for calculating their drags) even at very high Mach numbers. Nonlinear theory taken as standard of comparison is neither Busemann second-order nor shock-expansion but modified hypersonic small-disturbance relation suggested by Ivey and Cline [AMR **4**, Rev. 2579]. Charts based upon idealized base-pressure variation permit determination of optimum shape for given thickness.
Milton D. Van Dyke, USA

3153. Westley, R., The potential due to a source moving through a compressible fluid and application to some rotary derivatives of an aerofoil, *Coll. Aero. Cranfield Rep.* no. 54, 26 pp., 7 figs., Feb. 1952.

Paper concerns a theoretical study of the compressible flow resulting from a source in arbitrary motion through the fluid. Particular attention is given to the case of a helical path with applications to the calculation of rolling moment on an airfoil. Results obtained show that, for supersonic flight speeds, the rolling moment reverses sign at a particular Mach number for a given amount of sweepback. The solution is essentially an adaptation of the method of Liénard and Wiechert as applied to the electromagnetic theory of a moving point charge.
M. J. Thompson, USA

3154. Robinson, A., and Hunter-Tod, J. H., Bound and trailing vortices in the linearised theory of supersonic flow, and the downwash in the wake of a delta wing, *Aero. Res. Counc. Lond. Rep. Mem.* 2409, 14 pp., Oct. 1947, published 1952.

See AMR 2, Rev. 1419.

3155. Margolis, K., Some remarks on an approximate method of estimating the wave drag due to thickness at supersonic speeds of three-dimensional wings with arbitrary profile, *NACA TN* 2619, 9 pp., Feb. 1952.

Author discusses application and limitation of some semi-empirical profile corrections. Estimation is given for three-dimensional wings with various profiles by utilizing previously calculated drag coefficients. It is felt that judicious use of correction factors should yield satisfactory accuracy for many planform-profile combinations, especially at speeds for which the wing leading edge is supersonic.
Friedrich Keune, Sweden

3156. Squire, H. B., An example in wing theory at supersonic speed, *Aero. Res. Counc. Lond. Rep. Mem.* 2549, 16 pp., Feb. 1947, published 1951.

Using the approach suggested by Robinson [title source, 2548, 1946] for expressing the solutions to the linearized equations of motion in terms of the Lamé functions, author has demonstrated how to obtain the pressure distribution and related drag over thin nonlifting triangular wings with subsonic leading edges. Although the wings treated were basically of elliptic cross section in planes normal to the flow direction, the method used may be extended to include a larger class of thickness distributions, as was done by Heaslet and Lomax [AMR **5,** Rev. 821].

When computing the drag for these wings which have blunt (elliptic) leading edges, author points out that the linear theory may lead to incorrect results unless allowance is made for the large local pressures around the leading edges. A more complete discussion of these leading-edge forces may be found in a paper by R. T. Jones [AMR **4,** Rev. 1227]. Seymour Lampert, USA

3157. Hilton, W. F., Use of negative camber in the transonic speed range, *Aero. Res. Counc. Lond. Rep. Mem.* 2460, 2 pp., 6 figs., Mar. 1947, published 1952.

Certain difficulties are experienced when attempting free flight in the transonic speed range (0.8 to 1.2 of the speed of sound). These difficulties fall into two main classes, namely, (1) overcoming the somewhat high air resistance by means of improvement in engine design, and (2) balancing the aircraft for stable horizontal flight. The latter problem is considered here. Changes of trim are caused by sudden loss of wing lift in the transonic range, which decreases the downwash over the tail and possibly results in an uncontrollable nose-heavy dive. The use of negative wing camber for minimizing these effects is suggested, and the suggestion is found to be supported by wind-tunnel experiments.

From author's summary

3158. Robinson, A., and Hunter-Tod, J. H., The aerodynamic derivatives with respect to sideslip for a delta wing with small dihedral at zero incidence at supersonic speeds, *Aero. Res. Counc. Lond. Rep. Mem.* 2410, 12 pp., 3 figs., Dec. 1947, published 1952.

See AMR **2,** Rev. 1435.

3159. Tamada, K., On the hodograph method and analytic continuation of solution in the theory of compressible fluid, *Mem. Coll. Sci. Univ. Kyoto* (A) **26,** 21–30, 1950.

A method of analytically continuing solutions in the hodograph plane alternative to those of Cherry [*Proc. roy. Soc. Lond.* (A) **192,** 45–79, 1947], Lighthill [ibid. **191,** 352–369, 1947], and Goldstein, Lighthill, and Craggs [*Quart. J. Mech. appl. Math.* **1,** 344–357, 1948] is given, with particular reference to solutions which reduce to $\psi = \text{Im}\,(1 - qe^{-i\theta})^{\lambda}$ in the incompressible case. Author claims that his arrangement of the resulting series solutions is particularly good for computational purposes.

M. J. Lighthill, England

3160. Nichols, M. R., and Rinkoski, D. W., A low-speed investigation of an annular transonic air inlet, *NACA TN* 2685, 38 pp., Apr. 1952.

Paper deals with the application of fuselage scoops to a transonic airplane. From measurements conducted at low speeds on three fuselages ahead of the annular scoops, authors conclude that it is possible to maintain subsonic speeds ahead of the air inlets up to flight Mach number 1.2, provided that the forward part of the fuselage has proper proportions. Then, in reviewer's opinion, there must be a shock wave ahead of the fuselage. A boundary layer being absent there, boundary-layer troubles will not be met.

Low-speed measurements cover a wide range of inlet velocity ratios and angles of attack. Results are given in graphs.

J. G. Slotboom, Holland

3161. Chen, Y. W., Supersonic flow through nozzles with rotational symmetry, *Comm. pure appl. Math.* **5,** 1, 57–86, Feb. 1952.

Author considers problems of isentropic, irrotational, supersonic flow with axial symmetry which involve the singularity on the axis and which, therefore, cannot be treated by the classical method of characteristics. He examines particularly solutions determined by initial data given on two characteristic lines intersecting on the axis, and shows that the usual theorems concerning uniqueness and existence of such solutions apply in this case, provided that the radial component of velocity in the neighborhood of the point of intersection is at least of the order of the square root of the radial distance. The propagation of disturbances along such characteristic lines is also considered. The results are then applied to the design of perfect nozzles; here the general method developed can be simplified, and a solution is obtained in terms of convergent series. Maurice Holt, England

3162. Timman, R., Approximate theory of the oscillating wing in compressible subsonic flow for high frequencies, *Nat. LuchtLab. Amsterdam Rap.* F.99, 13 pp., 2 tables, 4 figs., Dec. 1951.

The problem of unsteady forces acting on an infinitely long oscillating airfoil in subsonic flow has been solved by several authors by reducing it to a boundary-value problem of the common wave equation and then employing Mathieu functions. To avoid the cumbersome numerical calculations of these series expansions for high frequencies, author uses the Kirchhoff diffraction formula, well known in optics and acoustics, to obtain an approximate solution. Numerical values for the aerodynamic derivatives are given for Mach numbers 0.3, 0.5, 0.7, 0.9, and $3 < \omega < 9$, when $\omega = l\nu/V$ (l is semichord of airfoil, ν circular frequency, V translational velocity).

Hans L. Oestreicher, USA

3163. Rabineau, B. A., Compressibility corrections for bodies of revolution, *J. aero. Sci.* **19,** 3, 197–200, 206, Mar. 1952.

An approximate formula is given for calculation of the perturbed velocity of a body of revolution at any subsonic speed from that of the corresponding body in incompressible fluid. The results of the present formula are compared with more elaborate calculations and experimental data. S. I. Pai, USA

3164. Wu, C.-H., and Brown, C. A., A theory of the direct and inverse problems of compressible flow past cascade of arbitrary airfoils, *J. aero. Sci.* **19,** 3, 183–196, Mar. 1952.

A unified approach to both problems is presented, based upon two experimental facts: (1) Variation in specific mass flow ρW, on mean cascade streamline closely follows the variation in channel width in pitch (y-) direction. (2) Shape of mean streamline follows that of mean channel line (or that of blade skeleton), but with lower curvature, and position of mean streamline is closer to suction surface than to pressure surface. This enables an approximate determination of flow on the mean streamline for both the direct and inverse problems. Equations of motion, continuity, and adiabatic state are then used to compute partial y-derivatives of W_z, W_y, and ρ up to second order in terms of known variations on mean streamline, leading to an extension of flow description in pitch direction by means of Taylor series. For high-solidity cas-

cades, three terms in the series are sufficient for engineering accuracy. Several applications are made, and accuracy of method is checked by finite difference calculations, showing excellent agreement.

In reviewer's opinion, this simple unified approach is a remarkable contribution to two-dimensional cascade problem. Its practical importance for turbomachinery design will depend upon further applicability of plane-cascade results to circular-blade arrangements. At present state of knowledge, it appears that further improvements in turboengine cascade performance are to be expected from a better insight of three-dimensional effects (radial equilibrium, secondary flows and losses, etc.) rather than from a further development of two-dimensional methods.

Pierre Schwaar, Switzerland

3165. Bitondo, D., and Lobb, R. K., The design and construction of a shock tube, *Inst. Aerophys. Univ. Toronto UTIA Rep.* no. 3, 27 pp., 18 figs., May 1950.

The shock tube consists basically of a long tube separated into two compartments by a diaphragm, across which pressure and internal energy are discontinuous. When this diaphragm is ruptured, a plane shock wave is propagated along the low pressure chamber. The tube lends itself to an easy method of studying nonstationary plane shock waves and stationary flow patterns for a wide range of Mach numbers.

The design and instrumentation for such a tube are presented in this report, and the factors affecting each are considered in detail. Suggestions for the improvement of techniques are made throughout the text. From authors' summary

3166. Westphal, W. R., and Dunavant, J. C., A compressible-flow plotting device and its application to cascade flows, *NACA TN* 2681, 21 pp., Apr. 1952.

To determine two-dimensional compressible flows through well-defined passages, use is made of plastic cams which automatically set the length-to-width ratio of rectangles formed by streamlines and equipotential lines represented by spring-steel wires. Reasonable agreement was obtained for the pressure distributions around four different turbine blade cascades and along the surface of a choked nozzle as determined by the reported method and by actual experiment.

Simon Ostrach, USA

3167. Craggs, J. W., The compressible flow corresponding to a line doublet, *Quart. appl. Math.* **10**, 1, 88–93, Apr. 1952.

Starting from standard elementary incompressible flows, Ringleb [*ZAMM* 1940] used the hodograph method to construct a number of exact compressible flows. Author considers singularities of the Ringleb solution corresponding to an incompressible doublet. In the physical plane, a limit line with a double cusp and four branches is found, and the flow thus extends over four sheets. Maurice Holt, England

3168. Johannesen, N. H., Experiments on two-dimensional supersonic flow in corners and over concave surfaces, *Phil. Mag.* (7) **43**, 340, 568–580, May 1952.

Two simple cases of two-dimensional supersonic flow were investigated by schlieren photography, viz., the flow in two consecutive corners and the flow over a circular arc profile concave to the stream. The flows were photographed for smooth and rough surfaces of the models, i.e., with laminar and turbulent boundary layers, and the observed flow patterns are compared with those predicted by inviscid theory. The flow with a turbulent boundary layer was in good agreement with inviscid theory outside the boundary layer, whereas the laminar boundary layer separated and a shock wave originated at the point of separation. In both cases, the agreement was good at the points far from the surface.

A discussion is given of the flow at the point of intersection of three shock waves, and of a shock wave and a centered compression. For the latter case, photographs show a weak shock wave in the direction of the downstream Mach line at the point of intersection.

From author's summary by A. Miele, Argentina

3169. Busemann, A., Application of transonic similarity, *NACA TN* 2687, 22 pp., Apr. 1952.

For steady inviscid transonic flow, the governing differential equation is essentially nonlinear, even if the velocity of a disturbance generated by a thin body is assumed small in comparison with the velocity of flight. In consequence, the principle of superposition becomes invalid and classical mathematics is of little help in achieving useful solutions. Under this difficult situation, von Kármán in 1946 applied dimensional reasoning to explore the over-all characteristics of an airfoil, and found the similarity behavior of the transonic flow in terms of appropriate dimensionless parameters called invariants. Since then, the transonic similarity law has been further explored and applied both theoretically and experimentally by a number of investigators. In addition to summarizing these results, this paper brings out a number of further interesting points about the transonic similarity, particularly on the number of possible invariants, the choice of the essential terms of the well-known von Kármán transonic equation, and the limitations on the choice of invariants according to the region of Mach number. Second, the additional interpretation and examples at various hypothetical values of the ratio of specific heats γ on the two-dimensional and axially symmetrical transonic flow are illustrative. Third, author mentions a very important point, that the Prandtl correction factor (more commonly known as the Prandtl-Glauert correction factor in the English language) for pure subsonic or supersonic flow can be applied to the potential flow and boundary-layer development as long as profile similarity is maintained. Author makes a further conjecture, that the transonic similarity rules can be extended to two more kinds of transonic flows: (1) Flow with negligible separation, and (2) profile with sharp convex corners where the separation points of the boundary layer are fixed in position. The above assertions of Prandtl and the author, however, seem subject to the restriction that the Reynolds numbers of the bodies must be kept invariant; otherwise, the transition point from laminar to turbulent boundary layer is uncertain. It should be remarked that the pressure coefficient c_p, unlike the lift, drag, or moment coefficients, should be considered a function of local position coordinates as well as a function of the nondimensional parameters as shown. Furthermore, for the validity of the derivation of the transonic invariants, the derivatives of the perturbation potential must exist, be nonzero, and be uniformly continuous up to the second order. Author does not mention this point in the text. Thus, the validity of the transonic similarity seems questionable for the airfoil with sharp convex corners where the velocity is discontinuous.

Author gives a clear statement of the range of thickness ratio of the validity of the transonic similarity law. It is not obvious whether this useful conclusion was drawn from experimental evidence or intuitional argument. At the boundary line of $\tau^2 = |1 - M^2|^2/(1 + \gamma)^2M^4$ in Fig. 5, it seems implausible that von Kármán's 5/3 power law of thickness ratio τ could jump to the unit power law of Prandtl so abruptly. Apparently, still further investigation is needed to find out if the exponent power changes gradually.

Chieh-Chien Chang, USA

3170. Kaye, J., Toong, T. Y., and Shoulberg, R. H., Measurement of recovery factors and friction coefficients for supersonic flow of air in a tube. 2—Results based on a two-dimensional flow model for entrance region, *J. appl. Mech.* **19,** 2, 185–194, June 1952.

See AMR **5,** Rev. 1827.

Turbulence, Boundary Layer, etc.

(See also Revs. 3192, 3233)

3171. Frenkiel, F. N., On the statistical theory of turbulent diffusion, *Proc. nat. Acad. Sci. Wash.* **38,** 6, 509–515, June 1952.

The turbulent diffusion from a continuously emitting point source moving at uniform mean velocity through the fluid is considered, with particular reference to the case of turbulent velocities of the same order of magnitude as the mean velocity. Assuming the turbulence to be homogeneous, isotropic, and nondecaying, an expression for the distribution of mean concentration is developed in terms of the single-particle Lagrangian correlation function introduced by G. I. Taylor. Simpler approximate forms are also given which are suitable for long and short diffusion times. It should be noted that the averaging process is described as being taken over a large number of particles emitted simultaneously from the source, whereas the mean concentration must be the average over a large number of trials at different times. This error has no effect on the final results.

A. A. Townsend, USA

3172. Roy, M., Reduction of drag and continuous suction of the boundary layer (in French), *Rech. aéro.* no. 25, 3–8, Jan.-Feb. 1952.

At the Brighton Conference (Sept. 1951), Sir Melville Jones and M. R. Head gave calculations that showed a saving of propulsive power of up to 50% for an airfoil when the boundary layer was removed by suction from a large part of the surface (assumed porous).

Roy shows that this saving is probably much overestimated, as it does not take into consideration the power required to effect the suction. The ejection of the air from the boundary layer through a slot in the trailing edge is discussed, as is also the use of this ejected air as part of the propulsive system. The effect of the size of the slot on the efficiency of the system is also considered.

As the calculations of Jones and Head and also those of Roy are based on parameters that have yet to be determined experimentally, the quantitative drag reduction due to the boundary-layer suction is still a matter of controversy. Only the case of an ideal wing of infinite span with no body interaction effects is considered. Although the fluid is assumed to have negligible compressibility, the calculations could easily be extended to include compressibility effects.

Ione D. V. Faro, USA

3173. Raspet, A., Boundary-layer studies on a sailplane, *Aero. Engng. Rev.* **11** 6, 52–60, June 1952.

Paper describes experiments made in flight with a T-G3A sailplane and consists of four phases: (1) Discussion of investigations made and other investigations made on the drag reduction by applying boundary-layer control through a trailing-edge slat; (2) comparison of the lift increases attained in flight to results obtained in other investigations; (3) discussion of several experiments made with various chordwise extents of porous suction and its effect on transition and stabilization of the laminar boundary layer; (4) discussion of methods of attaining automatic suction.

Boundary-layer control was applied to a 45 in. wide, 60-in. chord section of one wing. A spanwise suction slat was located at 95% chord. The porous surface was attained by pricking closely spaced holes in rows from 35% chord to 90% chord.

Investigation indicates no drag reduction by stabilization of the laminar boundary layer with boundary-layer control applied through a trailing-edge slot. Lift increases obtained were of smaller value than reported in other research (Regensheit) under similar conditions. Considerable drag reduction was obtained by retaining a laminar boundary layer to 95% chord with control applied through the porous surface. This was accomplished with low power requirements. It was found that the spacing of rows of holes in the chordwise direction was critical. Author plans continuation of the research to find porosity distribution for optimum suction quantities. Author recommends use of upper surface of wing at tip with a square tip at trailing edge for automatic suction.

Reviewer believes the results indicated by porous suction show great promise for extending the range of cross-country sailplanes, especially so if the power requirements can be reduced to values attainable with automatic suction.

Woodrow L. Cook, USA

3174. Loos, H. G., Some considerations on the boundary-layer interference at the connection of a wing and a wall (in Dutch), *Nat. LuchtLab. Amsterdam Rap.* A.1282, 115 pp., Feb. 1952.

The lift distribution at the wing root is influenced by the boundary layer of the fuselage. As the three-dimensional problem of viscous flow in the neighborhood of the wing root is not accessible to an analytical treatment, author modifies the problem: He studies the lift distribution of a wing near a wall (normal to the span of the wing) when the basic flow is a nonviscous parallel flow with a velocity diminishing toward the wall in such a way as the boundary-layer flow at the wall would demand. The vorticity of the basic flow is important for the computation of the lift distribution of a given wing.

First, author studies the *heavily* loaded wing in the boundary-layer flow of a flat plate with *small* velocity gradients (the basic flow actually does not decrease to zero at the wall). The starting point of this study is Hawthorne's theory of "secondary circulation in fluid flow" [see AMR **4,** Rev. 3607]. The well-known Prandtl equation for the lift distribution is modified due to the vorticity of the basic flow. Not being satisfied with the results of this study, author attacks the problem of the *lightly* loaded wing in a boundary-layer flow with *large* velocity gradients. He uses the von Kármán-Tsien theory [*Quart. appl. Math.* **3,** 1945] for wings in nonuniform flow. For a basic flow corresponding to turbulent boundary-layer profile of the fuselage (wall) $U = y^k$, several wing problems are studied in detail: Wing with constant chord between two walls; wing with constant chord placed from one wall to the other in a wind tunnel with rectangular section (only the boundary layer at the vertical walls of the tunnel is taken into account); wing combined with an infinitely long cylindrical fuselage. For the first of these three problems, numerical results are presented and qualitatively compared with Zwaaneveld's experiments [title source, A.1265].

I. Flügge-Lotz, USA

3175. Schubauer, G. B., and Klebanoff, P. S., Investigation of separation of the turbulent boundary layer, *NACA Rep.* 1030, 20 pp., 1951.

See AMR **4,** Rev. 352.

3176. Squire, W., On Broszko's theory of turbulent flow, *J. aero. Sci.* **18,** 7, 502–503, July 1951.

Critical comments are given on the derivation and consequences of Broszko's relations between the mean motion and the turbulent

fluctuations; these relations are used with the Reynolds equations. Author points out deficiencies of Broszko's theory, but states that, in spite of its deficiencies as a rational theory of turbulent flow, the theory gives satisfactory results for the mean velocity distribution. To show this, a calculated mean velocity distribution is compared with an experimental distribution for a pipe and for a channel, and the agreement is good.

Neal Tetervin, USA

3177. Chandrasekhar, S., On turbulence caused by thermal instability, *Phil. Trans. roy. Soc. Lond.* (*A*) **244**, 884, 357–384, Mar. 1952.

Theory presented here of turbulence in an incompressible fluid caused by the joint effects of gravity and a temperature gradient is based on the equations of motion, continuity, and heat conduction. The variation of density is taken into account only in so far as it modifies the effect of gravity. Further simplifications consist in neglecting the nonlinear terms in the equations of motion and heat conduction and in treating the turbulence as homogeneous and axisymmetric. Using the theory of axisymmetric vectors and tensors, a fundamental set of solutions of the equations governing stationary turbulence is obtained for the case when a constant mean adverse temperature gradient is maintained. From these solutions it follows that there will be a smallest size for the eddies, depending on the temperature gradient and the physical magnitudes. The field of turbulence can be analyzed into two modes characterized by the kinetic energy being confined, principally, to the vertical or horizontal direction.

Reviewer remarks that the velocity and temperature fluctuations will be small quantities of first order only when the temperature gradient maintained is not high. However, experiments indicated that convection changes from a cellular to a turbulent pattern for a much higher temperature gradient than is necessary to obtain thermal instability. It is, therefore, not known whether the neglect of the nonlinear terms in the equations of motion and heat conduction is justified from a physical point of view.

Julius Rotta, Germany

3178. Poisson-Quinton, Ph., The mechanism of boundary-layer control and its application to airplanes (in French), *Actes Coll. inter. Mécan.* **II**, *Publ. sci. tech. Min. Air, Paris*, no. 250, 133–156, 1951.

Recent progress in aeronautical technology has revived interest in boundary-layer control by suction and blowing. The jet engine seems well adapted to produce the air flow, and airplane configurations suitable for high-speed flight have deficiencies at low speeds which can be ameliorated by boundary-layer control. Present paper is general review of the mechanism of the effects of suction and blowing and of experiments conducted in France.

Author concludes that both suction and blowing are of interest for increasing lift only when they suppress or retard flow separation; the direct increase of lift when separation is not present is small. Hence, boundary-layer control is particularly applicable to maintenance of good flow over deflected trailing-edge controls. The applications described include blowing through a slot over ailerons and flaps with measurements of lift, drag, and pitching moment; the same configuration with a leading-edge flap with measurements in the trim condition; a combination of suction and blowing, the flow being induced by injection of compressed air on a wing having three hinge lines for nose flap, intermediate, and trailing-edge flap; and differential blowing over ailerons for lateral control. Engineers will wish to consult the original technical papers.

Much briefer accounts are given of experiments on applications to decrease drag and to control shock waves.

Hugh L. Dryden, USA

3179. Braslow, A. L., Burrows, D. L., Tetervin, N., and Visconti, F., Experimental and theoretical studies of area suction for the control of the laminar boundary layer on an NACA 64A010 airfoil, *NACA Rep.* 1025, 19 pp., 1951.

Supersedes papers reviewed in AMR **4**, Rev. 819; **3**, Rev. 2445.

3180. Stine, H. A., and Scherrer, R., Experimental investigation of the turbulent-boundary-layer temperature-recovery factor on bodies of revolution at Mach numbers from 2.0 to 3.8, *NACA TN* 2664, 20 pp., Mar. 1952.

A turbulent-boundary-layer temperature-recovery factor of 0.885 ± 0.011 was measured on both a 10°-cone and a 40°-cone cylinder at Mach numbers from 2 to 3.8 and Reynolds numbers based on surface cinematic viscosity from 4×10^5 to 4×10^6. Comparisons are made with available theories and experiments.

From authors' summary by J. R. Stalder, USA

3181. Nonweiler, T., Surface conduction of the heat transferred from a boundary layer, *Coll. Aero. Cranfield Rep.* 59, 14 pp., 3 figs., May 1952.

This note considers the effects of thermal conductivity upon the temperature distribution in the skin of a body (moving through air) due to the heat transferred from the boundary layer. It is found that the effects are of importance only very near the nose of the body, and that here the temperature reaches a maximum which, depending on the skin conductivity and thickness, may be appreciably less than the thermometer temperature, particularly at high speeds and altitudes of flight.

From author's summary by M. S. Uberoi, USA

Aerodynamics of Flight; Wind Forces

(See also Revs. 3158, 3173, 3174, 3179, 3200, 3201, 3219)

3182. Goodman, A., and Wolhart, W. D., Experimental investigation of the low-speed static and yawing stability characteristics of a 45° sweptback high-wing configuration with various twin vertical wing fins, *NACA TN* 2534, 25 pp., Nov. 1951.

Investigation was primarily an attempt to improve directional stability at low speeds of a high-speed aircraft configuration by replacing vertical tail on fuselage with vertical fins on the 45° sweptback wing. Tests were conducted in a 6- by 6-ft stability tunnel at a Mach number of 0.17 and a Reynolds number of 0.88×10^6. Results showed an improvement in directional stability at moderate and high angles of attack. Not much change was noted, however, in dihedral effect, damping in yaw, or static longitudinal stability characteristics.

Arthur L. Jones, USA

3183. Roy, M., and Le Bont, R., Turbo-reciprocating power plants. Application to aircraft propulsion (in French), *ONERA Publ.* no. 50, Part 1, 126 pp.; Part 2, tables; Part 3, diagrams, 1952.

Report presents elaborate study of possibilities of unconventional aircraft power plants, especially those consisting of a reciprocating engine operating with a turbocompressor. For purposes of comparative study, engine is taken in all cases as two-cycle fuel-injection type, with bore diameter of about 100 mm. Methods of calculation are discussed in great detail, and certain assumptions are admitted to be quite uncertain. It is concluded that following power plants offer some promise: (1) Plant in which reciprocating engine exhausts into turbine-driving supercharging compressor, with pressure ratios of about 3.6 at sea level and 7 at 36,000-ft altitude, and corresponding turbine-inlet gas temperatures of 400 C and 470 C; (2) plant in which compressor absorbs

entire output of engine, and all useful output is supplied by turbine shaft, with pressure ratios same as above and corresponding turbine-inlet temperatures of 400 C and 350 C; (3) plant in which engine drives low-pressure-ratio, high-flow compressor, most of compressor discharge air being by-passed around engine, heated, and discharged directly to atmosphere to furnish reactive thrust. Last plant can develop large power, but is heavier and more complicated than simple turbojet. Very detailed performance data for various power plants are given in parts 2 and 3, but, although sufficient for comparative purposes, these data do not have much generality. C. W. Smith, USA

3184. Sternfield, L., Some effects of nonlinear variation in the directional-stability and damping-in-yawing derivatives on the lateral stability of an airplane, *NACA Rep.* 1042, 9 pp., 1951.

See AMR **4,** Rev. 1282.

3185. Marx, A. J., and Buhrman, J., An analysis of the pitching motion of an aeroplane due to sideslip, *Nat. LuchtLab. Amsterdam Rap.* V.1602, 6 pp., 1951.

Using certain simplifying assumptions, the problem is represented by four equations of motion. Most of the necessary stability derivatives are determined from wind-tunnel tests, and the others are calculated from approximate formulas. Equations are solved by numerical step-by-step integration. Method is applied to two aircraft and results are compared with flight-test results. Dana Young, USA

3186. Meyer, J. R., Jr., An investigation of bending-moment distribution on a model helicopter rotor blade and a comparison with theory, *NACA TN* 2626, 91 pp., Feb. 1952.

This valuable paper presents measurements of blade-bending moments by means of strain gages on a three-blade model helicopter 5 ft in diam, under hovering and simulated forward-flight conditions. Both hinged-blade and fixed-at-root-blade configurations were tested up to advance ratios of 0.50 and 0.90, respectively. Curves of maximum bending-moment distribution and harmonic bending moments are presented showing a large contribution from the fifth-harmonic sine coefficient in the case of hinged-blade configuration. (Reviewer's note: Similar results have been obtained recently by Cornell Aeronautical Laboratory.)

A comparison is made between experimental results and theoretical calculations using the Goodyear tabular-dynamic method and the De Guillenchmidt method. A modification of the Goodyear method appropriate to the fixed-at-root blade is given. The agreement between theory and experiment is, in general, not very good, indicating that present-day methods of computing blade-bending moments are far from satisfactory. Author points out that the discrepancy is due to the neglect of the higher harmonics of the aerodynamic loading and to the simplifying assumptions made in present-day helicopter theory. Heinz Parkus, USA

3187. Bartholomew, F. E., and Marshall, S. D., The determination in flight of the body drag and the mean blade profile drag coefficient of a helicopter, *Coll. Aero. Cranfield Rep.* 56, 10 pp., 6 figs., Apr. 1952.

Relationships for power required in forward flight are modified to permit plotting results of partial climbs as two straight lines. Slopes of these lines give body drag of helicopter and mean profile drag of the blades, respectively. Body drags thus obtained seem to be quite accurate, but blade profile-drag coefficients are less reliable. However, authors believe that correct results can also be achieved here if transmission and tail-rotor losses are assessed to an accuracy of 1%. W. Z. Stepniewski, USA

3188. Fiszdon, W., Influence of "dry" friction in the control system on the lateral stability of aircraft (in Polish), *Techn. Lotn.* **7,** 2, 40–43, Mar./Apr. 1952.

Paper represents a résumé of works of Greenberg and Sternfeld [AMR **1,** Rev. 194] and Neumark (*Aero. Res. Counc. Lond. Rep. Mem.* 2259) on lateral oscillations of an aircraft with rudder free, including the effect of friction in the control system. After developing general equations of motion, author discusses the dry friction and its effect on stability considerations. Derivation of well-known conclusions (see the mentioned original papers) closes the note. M. Z. Krzywoblocki, USA

3189. Mitchell, K., Thorpe, A. W., and Frayn, E. M., A theoretical investigation of the response of a high-speed aeroplane to the application of ailerons and rudders, *Aero. Res. Counc. Lond. Rep. Mem.* 2294, 92 pp., 1945, published 1952.

Paper presents tables of stability roots and curves showing lateral response of aircraft to sharp-edged gust and to control movements for a wide range of stability parameters. Solutions were obtained on differential analyzer. A. H. Flax, USA

3190. Morikawa, G., Supersonic wing-body-tail interference, *J. aero. Sci.* **19,** 5, 333–340, May 1952.

Nonlinear difficulties in wing-body-tail interference are avoided by restricting the study to angle-of-attack near zero for in-line configurations where wing and tail lie in the same horizontal plane. Comparison of interference due to two limiting representations of this flow field behind lifting wings—vortex sheet and vortex line—shows the same qualitative behavior for limiting cases of the design parameters. These extreme parameters correspond effectively to zero aspect ratio and infinite aspect ratio of either surface.

Author considers only delta planforms and circular cylindrical bodies, but states the possibility of extension to more general cases. He concludes that his methods may be applied to predict the behavior of configurations in which the tail is not in the same plane as the wing, and that further analysis for intermediate values of wing and tail aspect ratio does not appear warranted by currently available experimental data. Holt Ashley, USA

3191. Zaat, J. A., Pressure distribution calculations with viscosity corrections for aerofoils with flaps, *Nat. LuchtLab. Amsterdam Rap.* F.88, 20 pp., 5 tables, 6 figs., Nov. 1951.

The method of conformal mapping is used for calculating pressure distribution on flapped airfoils with a large wake region behind the flap. The treatments are based on the works of Theodorsen [*NACA Rep.* 452], Pinkerton [*NACA Rep.* 563], and Walz [*Jahrb. Deutsch. Luftfahrtforsch.* p. I, 265, 1941]. The mapping is made for the original profile shape with an extended tail part formed about the wake region. Through an iteration process, the Kutta condition is performed in such a way that the tail part gives no lift.

The procedure is described in all details for the NACA 23012 airfoil with split flap. Good agreement is found with measured pressure distribution. N. Scholz, Germany

3192. Hoerner, S. F., Aerodynamic drag. Practical data on aerodynamic drag, Midland Park, New Jersey; Author, 1951, vii + 259 pp. $5.50.

This is more than a handbook. The numerous formulas and graphs are discussed in sufficient detail so that they can be used intelligently. Some of the data presented, particularly those measured in Germany, were not readily available before. The completeness with which the subject of aerodynamic drag is

covered is indicated by the chapter titles: General; Skin-friction drag; Pressure drag; Drag of surface irregularities; Drag of streamline bodies and sections; Drag due to lift; Interference drag; Drag of aircraft components; Drag of vehicles; Drag of internal-flow systems; Subsonic influence of compressibility on drag; Supersonic drag; Drag in rarefied gases.

Gerald Nitzberg, USA

3193. Suydam, H. B., Hydrodynamic characteristics of a low-drag, planing-tail flying-boat hull, *NACA TN* 2481, 20 pp., Feb. 1952.

Model tests were made to compare hydrodynamic characteristics of a flying boat with two hull configurations: conventional vs. low-drag planing-tail type. Tests indicated that planing-tail hull was superior with respect to elevator-deflection range, center-of-gravity location, porpoising, landing stability, planing resistance, and load-resistance ratio. Although static trim of planing-tail hull was higher, variation of trim with speed during take-off was smaller than for conventional hull. The test results are presented in graphical form.

From author's summary by T. F. O'Brien, USA

3194. Stone, R. W., Jr., Estimation of the maximum angle of sideslip for determination of vertical-tail loads in rolling maneuvers, *NACA TN* 2633, 46 pp., Feb. 1952.

This report studies vertical tail loads in roll maneuvers of two airplane designs representing, respectively, current and World War II airplanes. Previous methods of estimating vertical tail loads in roll maneuver assumed characteristics and limitations about the aerodynamic derivatives and the inertia of the airplane that are no longer applicable in current design, which features mass distributions having low rolling inertia. Solution of the nonlinear equations gives higher maximum sideslip angles for current airplanes, particularly when the airplane is in pitching motion. Linearized equations, with product of inertia terms included, appear to be adequate in the absence of pitching. Effects other than those of maximum sideslip, such as on period damping, and pitching motion, are noted and illustrated by graphs. Current designs have markedly lower sideslip angles but higher loads because of the higher speeds, and, therefore, greater accuracy in calculation is required. Information about the step-by-step integrations for the nonlinear system is presented and results are compared in tabular form with the results found by the several known linear methods.

M. G. Scherberg, USA

3195. Falkner, V. M., The effect of pointed tips on wing loading calculations, *Aero. Res. Counc. Lond. Rep. Mem.* 2483, 2 pp., 1 table, 1 fig., Oct. 1946, published 1952.

The standard formulas for calculation of wing loading by vortex lattice theory, which involve a sequence starting with $(1 - n^2)^{1/2}$, are not strictly valid for pointed wing tips. On the other hand, the use of a sequence starting with $1 - n^2$ leads to over-correction. By considering the mean of these two solutions for a delta wing, it is shown that the error introduced by the use of the standard sequence is small and is on the safe side as regards bending moments.

From author's summary

3196. Gaugh, W. J., and Slap, J. K., Determination of elastic wing aerodynamic characteristics, *J. aero. Sci.* **19**, 3, 173–182, Mar. 1952.

Paper investigates an iterative solution of the integral equation governing the aeroelastic deformation of a wing. Convergence difficulties for the case of swept wings are overcome by a process of analytic extension. Applications to problems of loading and stability are worked out in some detail, and comparison with a wind-tunnel test shows remarkably good agreement. Basic assumptions for both structures and aerodynamics would seem to limit applications to cases of reasonably large aspect ratios.

W. H. Hemp, England

3197. Zalovcik, J. A., Summary of stall-warning devices, *NACA TN* 2676, 15 pp., May 1952.

The principles involved in the operation of several types of stall-warning devices are described, and conditions under which difficulty may be experienced are pointed out. In the discussion, stall-warning devices are grouped as special stall-sensing devices and angle-of-attack-sensing devices. Methods of transmitting the warning to the pilot are also discussed. Some specific examples of stall-warning devices are illustrated and described.

From author's summary

3198. Teisseyre, J. H., Calculation of lift distribution along the wing span (in Polish), *Techn. Lotn.* **7**, 2, 32–39, Mar./Apr. 1952.

Paper contains the translation of methods of Glauert, Lotz, some papers of *NACA*, etc., known for one generation, on calculating the lift distribution along the span of finite wings. The following cases are reproduced: Elliptic loading, elliptic planform, general single wing, Fourier series method, rectangular and trapezoidal planforms. The reproduction is very good, with all the details of numerical calculation.

M. Z. Krzywoblocki, USA

3199. Helliwell, J. B., The automatic control of an aeroplane in the landing approach onto an aircraft carrier, *Aero. Quart.* **3**, part 4, 241–262, Feb. 1952.

Paper investigates the possibility of the completely automatic longitudinal control of an aircraft in the deck-landing approach. The existence of a radio-beam system transmitted from the carrier is presupposed, and a study is made of the motion of the aircraft down this beam under automatic control. The motion of the carrier is assumed to consist of a steady forward velocity modified by a nonpitching sinusoidal oscillation in the vertical plane. The most successful method of approach control proposed is based on the use of a single beam emanating from a source at a fixed distance above the carrier deck and fixed in direction. Other methods of control are also investigated.

From author's summary by W. Oppelt, Germany

Aeroelasticity (Flutter, Divergence, etc.)

(See also Revs. 3075, 3131, 3162, 3196)

3200. Lilley, G. M., An investigation of the flexure-torsion flutter characteristics of aerofoils in cascade, *Coll. Aero. Cranfield Rep.* 60, 49 pp., 22 figs., May 1952.

In part 1, author describes results from a series of flexure-torsion flutter tests of airfoils arranged in cascades. The airfoil removed from the cascade and placed in an infinite fluid is called the "isolated airfoil." Compared to that of the isolated airfoil, the critical cascade-flutter speed is found to decrease and the flutter frequency to increase as the gas-chord ratio is reduced. Stagger is reported to have little effect on flutter. During flutter, adjacent airfoils oscillate approximately 180° out of phase.

In part 2, author gives an approximate analysis of cascade flutter. From the observed antiphase of adjacent blades, he concludes that the individual airfoil in the cascade acts like one isolated between parallel plates. For these, he rederives essentially the results of Timman [AMR **5**, Rev. 766], to whose work frequent reference is made. Furthermore, author says that it can

be shown that the conditions for single-degree-of-freedom flutter cannot be satisfied.

The analytical part lacks the generality of the experiments because antiphase of adjacent blades is a constraint of the theory, but not of the experiments. The experiments demonstrate approximate antiphase of adjacent blades as one of the results, not a condition, of the problem. Furthermore, since fluttering in pitching oscillations is known to exist in isolated airfoils [Smilg, B., AMR **3**, Rev. 2461], it appears unlikely that the conditions for its existence cannot be satisfied for airfoils between parallel plates.

R. M. Rosenberg, USA

3201. Radok, J. R. M., Dynamic aeroelasticity of aircraft with swept wings, *Coll. Aero. Cranfield Rep.* 58, 42 pp., 6 figs., Apr. 1952.

General integrodifferential equations of motion covering the rigid body motions and all relevant deformations of fuselage and sweptback wings (referred to oblique coordinates) of the aircraft are established. The wing elasticity is treated by the method of Hemp ["Theory of the uniform two-cell swept box," Lectures given at the Coll. of Aero. Cranfield, 1951]. The nonstationary aerodynamic forces are represented by strip theory corrected to steady-state spanwise distribution. The vibrations, flutter, dynamic stability, and the gust case are considered. Author hopes to have selected notations ascertaining both great clarity and conciseness, but reviewer doubts whether his system and methods are entirely satisfactory in these respects. The paragraph on methods of approximate solution is superficial; the case where polynomial or measured modes are used requires additional consideration.

J. H. Greidanus, Holland

3202. Broadbent, E. G., and Kirkby, W. T., Control surface flutter, *J. roy. aero. Soc.* **56**, 497, 355–375, May 1952.

Paper presents an excellent qualitative discussion on the general characteristics of tab and control surface flutter at subsonic speeds. Conclusions are typical of U. S. experience, and nothing radically new is presented. Paper should be valuable to newcomers in the field and for flight-test and design engineers. A realistic discussion on the philosophy of flight testing is set forth.

C. Desmond Pengelley, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 3014, 3164, 3183, 3186, 3221)

3203. Cohen, L., Theoretical investigation of velocity diagrams of a single-stage turbine for a turbine for a turbojet engine at maximum thrust per square foot turbine frontal area, *NACA TN* 2732, 34 pp., June 1952.

High thrust per unit frontal area is obviously desirable if good efficiency can be maintained. Author assumes thrust per unit area is maximum when, with conditions otherwise fixed, weight flow is maximum for specified shaft power output required to drive compressor. Weight flow is taken as maximum when axial component of velocity on downstream side of rotor is sonic, and absolute exit whirl is minimum. Results are presented in form of velocity diagrams for various combinations of turbine-inlet temperature and blade speed. Constant centrifugal stress at root of blades and fixed flight conditions are assumed; corrections are introduced for turbine efficiency. Results indicate that, in general, relative velocities must be supersonic on both leading and trailing sides of blades, and that diagram at mean radius is of impulse type. Report gives exploratory picture, but does not purport to give results attainable in practice, since usual design limitations are ignored in addition to other approximations.

C. W. Smith, USA

3204. Lieblein, S., Theoretical and experimental analysis of one-dimensional compressible flow in a rotating radial-inlet impeller channel, *NACA TN* 2691, 47 pp., Apr. 1952.

An analysis of the one-dimensional compressible flow in a rotating radial-plane impeller channel was conducted in order to provide an insight into the characteristics of the passage mean flow under the influence of centrifugal forces and losses. From a theoretical investigation of the flow in an illustrative impeller channel with convergent-divergent area variation, the behavior of the flow along the channel was found to be similar in trend to the flow in a stationary convergent-divergent nozzle. The critical (sonic) section of the rotating channel occurred upstream of the geometric throat. The effect of the losses on the flow was similar to the effect of a reduction of the flow area.

A further one-dimensional analysis was conducted of the flow in an experimental radial-inlet impeller containing static-pressure taps along the stationary front shroud. The behavior of the mean flow along the impeller passage was generally similar to that of the flow along a rotating radial channel in which the effective flow area in the inlet region varied with the operating point.

From author's summary by W. E. Moeckel, USA

3205. Vallander, S. V., Flow of liquid in a turbine (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **84**, 4, 673–676, June 1952.

Author assumes (1) frictionless, incompressible flow; (2) the absolute motion in the turbine is irrotational and the stream surfaces are axially symmetric; (3) the infinitesimal distance between two stream surfaces changes along the meridional streamlines so little that its influence on the equation of continuity may be neglected. Under these assumptions, the analysis of the flow in a Kaplan turbine is reduced to the Cauchy-Riemann equations for two-dimensional motion and to the solution of the Dirichlet and Neumann problems.

Reviewer remarks: (a) Assumption (2) contradicts the real conditions in the Kaplan turbine, since the flow in the transition space in the usual types of construction is an intense vortex flow [see AMR **5**, Rev. 2152]; (b) the assertion that the action of the runner blades is equal to the action of an infinite two-dimensional airfoil cascade is wrong. It is known that the hydrodynamic system of a Kaplan runner consists of a system of one-sided infinite, bound vortex lines, which are distributed continuously on the mean surface of the blade. The blade thickness is taken into consideration by means of sources and sinks, situated on the mean surface of the blade. The three-dimensional character of the flow in the turbine is of decisive importance both for the blade form and for the characteristic curves of the turbine [Cf. Strscheletzky, "Hydrodynamische Grundlagen zur Berechnung der Schiffsschrauben," Braun-Verlag, Karlsruhe, 1951, and "Berechnung der Schaufelform von Kaplan-Turbinen," *ZAMM*, 8/9, 1952]. Hence, assumption (3) is hardly permissible for the usual construction form of the Kaplan turbine.

M. Strscheletzky, Germany

3206. Lal, J., Characteristic curves of an impulse turbine under loading and braking, and efficiencies under load [**Die Kennlinien einer Freistrahlturbine im Triebgebiet sowie im Bremsgebiet und die Wirkungsgrade im Triebgebiet**], Wien, Springer-Verlag, 1952, 118 pp. $3.50.

Book is result of thesis work carried out at ETH, Zürich, Switzerland. It covers theoretical and experimental investigation of an impulse turbine (263-ft head, 1285 gpm, 750 rpm, 70 hp, 20 buckets, mean wheel diam 17.7 in.) outside its normal performance range, namely, $u < 0$ and $u > u_{\max}$ where turbine acts as a hydraulic brake. For quick deceleration of turbines after shutdown, hydraulic brakes are in use which may consist of a second

smaller wheel with a separate nozzle (normal operating range $0 \leqq u \leqq u_{max}$), or of a separate nozzle that works against the underside of the buckets of the main wheel ($u < 0$). Braking also occurs when the wheel is propelled externally and runs with a higher speed than u_{max}. In this case, the buckets cut into the jet and accelerate the water particles. A brief analytical treatment is given of ideal and real flow through an impulse turbine, change of torque and power for the operating ranges at constant head and nozzle opening, and the efficiencies at varying heads. The test setup and procedures are excellently described in great detail, and the results are presented in graphical and tabulated form.

With the jet impinging on the backside of the buckets, the maximum efficiency obtained was 10.6% at a needle opening of 37.5%, as compared with 81.6% at normal operation. By redesigning the backside of the buckets, this figure could be improved. From the experimental results, the smallest jet diameter for a separate brake nozzle to work into the buckets is determined, as well as dimensions for a separate brake wheel and nozzle. In a large pumped-storage installation, such a brake wheel can be used to stop the main wheel and subsequently start the pump coupled with the generator (running as motor) in the opposite direction. When synchronous speed is reached, the pump is switched to electric drive.

The book presents a wealth of performance data which will be of great interest to anyone working in the turbine field.

O. E. Teichmann, USA

3207. Dandois, M., and Novik, D., Application of linear analysis to an experimental investigation of a turbojet engine with proportional speed control, *NACA TN* 2642, 38 pp., Feb. 1952.

Results of an analytical and a sea-level experimental investigation of a turbojet engine with proportional speed control are presented. Linear analysis and description of the engine as a first-order linear system proved adequate for analytical prediction of the response and the stability of the controlled engine, although instability calculations were found to be much more critical than response calculations.

On the basis of a compromise between speed of response and oscillations, an optimum loop gain was found. Increased loop gain increased the speed of response and decreased the speed error, but ultimately led to instability characterized by an essentially constant frequency and constant amplitude oscillation. Operation near the limits of stability required a decrease in control gain with decreasing engine speed.

From authors' summary by J. G. Slotboom, Holland

3208. Stephan, W., Design of Föttinger torque converters (in German), *ZVDI* **94**, 2, 50–58, Jan. 1952.

Paper gives a good general review of practical applications of hydrodynamic transmissions and their characteristics, illustrated by figures and diagrams. "Föttinger coupling" and "Föttinger transformer" are especially dealt with as basic types. Combinations of hydrodynamic transmissions and mechanical gears, used particularly for motor cars, are described.

Theoretical considerations are mentioned only briefly. Damping effect of Föttinger transmissions with regard to torque vibrations is cited briefly by means of two diagrams showing experimental investigations.

The reader interested in more details will find a large bibliography at the end of the paper. E. Mühlemann, Switzerland

3209. Brajnikoff, G. B., Method and graphs for the evaluation of air-induction systems, *NACA TN* 2697, 41 pp., May 1952.

A method of determining an optimum air-induction system for a jet engine is described, and graphs are presented for a rapid evaluation of net thrust coefficients in a variety of jet-propulsion systems. An optimum induction is considered to be that which would produce flow through the inlet equal to engine requirement at given operating conditions. The analysis is based on the following idealizations: (1) Working substance behaves essentially as a perfect gas with constant specific heats and a constant molal mass. (2) Pressure drop between compressor and turbine is neglected. (3) Combustion is complete. (4) Flow in exhaust nozzle is isentropic.

The method is illustrated by examples of a ramjet and a turbojet, and the calculations are systematized and presented in the form of well-organized computing tables.

Antoni K. Oppenheim, USA

3210. Marble, F. E., and Michelson, I., Analytical investigation of some three-dimensional flow problems in turbomachines, *NACA TN* 2614, 109 pp., Mar. 1952.

Paper collects further useful contributions to linear theory of axially symmetrical flow of incompressible, inviscid fluid in axial-flow turbomachines [see also, Marble, AMR **1**, Rev. 1400]. Linearization means that radial velocity and deviation of axial velocity from mean through-flow velocity are considered small, and are calculated to first-order approximation. Axially symmetrical solution (infinite number of blades) provides corrections accounting for most of three-dimensional effects neglected by cascade theory, especially for low pitch-chord ratios.

Theory distinguishes three (additive) components of flow, viz., (1) uniform through-flow, (2) radial equilibrium solution, correct for upstream and downstream of the blade row, (3) fine-structure accounting for radial accelerations. Tables of an influence function, for hub ratio 0.6, permit rapid evaluation of fine structure by punch-card method: two numerical examples are given.

A simple, actuator-disk approximation to fine-structure solution is applied, with a numerical example in each case, to the discussion of (1) transients in the first few stages of a multistage machine, (2) fluctuations of the axial velocity distribution within the deeply embedded stages [see also Wu and Wolfenstein, AMR **4**, Rev. 1316], (3) performance, in its dependence on the radius and on the aspect ratio, of a single-blade row with prescribed distribution of flow angle at trailing edge; (4) off-design performance of a blade row. In addition to their immediate interest, the examples provide a useful introduction to how theoretical knowledge of the fine-structure solution can be employed to obtain simple, approximate answers to practical questions.

A modified linear theory is proposed for the case of a through-flow velocity varying with the radius. The linearized solution is also given for a machine such that hub and casing are coaxial cones with common vertex. R. E. Meyer, England

Flow and Flight Test Techniques

(See also Revs. 3193, 3197)

3211. Moore, F. K., Use of the boundary layer of a cone to measure supersonic flow inclination, *NACA TN* 2723, 21 pp., June 1952.

The instrument measures the difference of total pressure recorded by two probes pointing toward the apex of the cone. Results of a single test indicate that sensitivity is limited to angles of attack less than the cone semivertex angle.

From author's summary by A. Petroff, USA

3212. Pearson, C. E., Measurement of instantaneous vector air velocity by hot-wire methods, *J. aero. Sci.* **19**, 2, 73–82, Feb. 1952.

Details are given of the theory, construction, and use of a hot-

wire anemometer for measuring instantaneous and average air velocities. For Mach numbers less than 0.3, author claims average velocities are measured with accuracy of better than 2%, and instantaneous velocities with accuracy of 10%.

A correction method for taking into account oxidation and plastic flow of the wire is included. Experimental data are presented to show that use of correction equations eliminates need for calibrating hot-wire anemometers more than once. Correction method should be of value to other investigators in this field. S. F. Gilman, USA

3213. Raab, L., A direct indicating anemometer, *Ark. Geofys.* **1**, 2/4, 117-121, 1951.

Anemometer contacts maintain a voltage across an L-C circuit which is dependent upon contact frequency (and thus wind speed). Circuit diagram and calibration curves are shown. Similar system was devised by U. S. Weather Bureau a number of years ago [see Middleton, "Meteorological instruments"]. Ferguson Hall, USA

3214. Fish, R. W., and Parnham, K., Focussing Schlieren systems, *Aero. Res. Counc. Lond. curr. Pap.* 54, 14 pp., 11 figs., Nov. 1950, published 1951.

Paper discusses in simple geometric terms the optics of focusing schlieren systems which differ from conventional ones by the use of a pair of complementary grids in lieu of slit source and knife edge. The schlieren head is thus used at a much smaller effective *f*-number and loses correspondingly in depth of focus. On the screen there appears a sharp schlieren image of the plane conjugate to the former with respect to the schlieren head, overlaid by the smeared-out effects of inhomogeneities not in focus. Quantitative evaluation is therefore impossible, and such arrangements serve principally to localize major disturbances in an otherwise not too irregular field. Paper concludes with rough optical specifications for three specific focusing systems which are individually analogous to the principal types of conventional schlieren arrangements: Single schlieren head with direct illumination, single schlieren head with lens-formed source of illumination, and parallel beam system. F. J. Weyl, USA

3215. Criborn, C. O., Determination of pressure-time curves of the shock wave by a new method, *Appl. sci. Res.* (*A*) **3**, 3, 225-236, 1952.

Author describes a "corona pressure gage" for measuring static pressures. Instrument consists of a ring electrode (about 8-mm diam) and a central needle cathode. Current between these electrodes during "corona" discharge is a function of the static pressure with small corrections for humidity, temperature, and misalignment. Its use is demonstrated by static pressure-time records of a shock wave from detonating TNT and of the flow in a vortex. F. G. Blight, Australia

3216. Rosenberg, B., The use of doubly refracting solutions in the investigation of fluid flow phenomena, *David W. Taylor Mod. Basin Rep.* 617, 39 pp., Mar. 1952.

Report surveys the literature concerning double refraction of colloidal solutions and of pure liquids. The feasibility of using doubly refracting liquids for qualitative and quantitative flow studies is discussed. The desirable quantities of Bentonite are stressed. The optical theory is presented. An apparatus is described that is under construction for the David Taylor Model Basin, to be used in studying flow phenomena using Bentonite solutions. Methods are suggested for obtaining pressure distribution, viscous drag, and viscous lift after velocity distribution and streamlines have been obtained. Experimental data are not offered to confirm the theoretical treatment that is presented. Glen N. Cox, USA

3217. Crump, E. S., A new method of gauging stream flow with little afflux by means of a submerged weir of triangular profile, *Proc. Instn. civ. Engrs.* **1**, 2, 223-242, Mar. 1952.

The high cost of single-gage measuring weirs, resulting mainly from the large afflux required to insure modularity, is often found to be prohibitive. (When the discharge of a weir depends only upon the upstream head and is independent of the downstream head, its performance is said to be modular.) To substantially reduce afflux and cost, a measuring weir must be capable of operating when submerged (i.e., in the nonmodular range), and this involves double-gaging with its attendant complexities and liability to error. Author proposes a new method of double-gaging applied to a weir of triangular profile that overcomes these effects. The upstream gage records the total head T by means of a Pitot pier; the downstream gage records the pressure head P at a point on the downstream face of the weir a short distance from the crest. Tests made by the author on a model weir show that the modular coefficient of discharge is sensibly constant, and that the ratio of actual-to-modular discharge (for a given value of T) is a unique function of the ratio P/T of the two gage readings. A table presenting this unique relationship provides a ready means of calculating the discharge from recorded values of T and P. From author's summary by A. Schlag, Belgium

3218. Elrod, H. G., Jr., and Fouse, R. R., An investigation of electromagnetic flowmeters, *Trans. ASME* **74**, 4, 589-594, May 1952.

See AMR **5**, Rev. 1529.

3219. Slaymaker, S. E., Lynn, R. R., and Gray, R. B., Experimental investigation of transition of a model helicopter rotor from hovering to vertical autorotation, *NACA TN* 2648, 29 pp., Mar. 1952.

Measurements of the sinking velocity of a model helicopter were made during the transition from hovering to steady autorotative vertical descent. Unexpectedly high rates of descending vibration were encountered which were not predicted. The high rates of descent are attributed to the model falling into its own downwash. Models released at conditions of zero thrust at autorotative speed reached vertical velocities predicted by theory. This indicates that the initial conditions are important in predicting the performance in the transition range.

Results are presented for variations in disk loading, blade angle, and rate of change of blade pitch. Robert C. Kidder, USA

Thermodynamics

(See also Rev. 3255)

3220. Himpan, J., On a new equation of state of gases (in French), *C. R. Acad. Sci. Paris* **234**, 26, 2523-2525, June 1952.

An equation of state is proposed similar to that of Van der Waals but having four constants instead of two

$$p + a/T^nV(V - b) = RT/(V - c)$$

The constants a, b, c, n can be expressed in terms of the critical temperature, pressure, and specific volume, and the Boyle temperature and the equation can be put in reduced form as an expression of a modified "law" of corresponding states. Based on a comparison with P-V-T data for CO_2 near its critical temperature and H_2 at zero and 65 C, the equation is shown to be su-

perior to those of Wohl and of Beattie-Bridgman, having three and five constants, respectively. R. L. Pigford, USA

3221. **Berman, K., and Logan, S. E., Combustion studies with a rocket motor having a full-length observation window,** *J. Amer. Rocket Soc.* **22,** 2, 78–85, 103, Mar.-Apr. 1952.

A method is described for studying combustion in rocket motors by means of high-speed photographs taken through a longitudinal quartz window, cooled by nitrogen. From the observed time-vs.-distance pattern traced by radiation sources, conclusions are drawn regarding gas-velocity distribution and velocity and pressure fluctuations in the engine. For three different injector-head types, test data are presented and analyzed for ranges of stable and unstable combustion.

Andrew Fejer, USA

3222. **Strickland-Constable, R. F., The burning velocity of gases in relation to the ignition delay period,** Third Symp. Combust. Flame Expl. Phenom.; Baltimore, Md., Williams & Wilkins, 229–235, 1949. $13.50.

Paper deals with propagation of flame in a combustible mixture. If heat propagates only by conduction and flame speed is known, then temperature distribution in front of flame can be computed. Additional assumption is necessary to determine occurrence of ignition. Author introduces, instead of a fixed ignition temperature, a concept which allows time for ignition reactions to take place. At fixed temperatures of the mixture, this time can be observed in the form of an ignition delay which depends exponentially upon temperature. For the varying temperatures in front of the flame, author describes the "state of activation" by an integral (time spent at low temperatures contributes less to the integral than time at high temperatures). Ignition occurs when integral assumes the value one. One numerical example is carried out and experimental evidence is discussed.

Gottfried Guderley, USA

3223. **Williams, D. T., and Bollinger, L. M., The effect of turbulence on flame speeds of Bunsen-type flames,** Third Symp. Combust. Flame Expl. Phenom.; Baltimore, Md., Williams & Wilkins, 176–190, 1949. $13.50.

The turbulent flame speeds of propane, ethylene, and acetylene are measured in a Bunsen burner at Reynolds numbers of 3000–35,000. An empirical formula connecting the ratio of turbulent to laminar flame speeds with tube diameter and Reynolds number is given. The experimental results disagree with the theories of Damköhler and Schehlkin.

Authors recognize arbitrariness in selection of turbulent flame surface from their photographs. Comparison should be made with later work of Karlowitz [*J. chem. Phys.* **19,** 5, 541–547, May 1951].

William Squire, USA

3224. **Dugger, G. L., Effect of initial mixture temperature on flame speed of methane-air, propane-air, and ethylene-air mixtures,** *NACA Rep.* 1061, 12 pp., 1952.

Supersedes paper reviewed in AMR **4,** Rev. 2230.

3225. **Altseimer, J. H., Photographic techniques applied to combustion studies—two-dimensional transparent thrust chamber,** *J. Amer. Rocket Soc.* **22,** 2, 86–91, 103, Mar.-Apr. 1952.

Lucite walls spaced 0.47 inch apart permitted motion pictures at 2000 to 3000 frames per sec of combustion with each of five different injector types in a rocket thrust chamber 16 in. long over-all, 4.375 in. high in the combustion space, and having a chamber-to-throat area ratio of 5.9. The four injectors of repetitive pattern design produced, to a greater or less degree, flames with longitudinal striations that persisted the length of the combustion space. In-phase fluctuations of flame intensity and chamber pressure were observed in four of thirteen runs with a fifth injector. Combustion eddies were tracked to produce plots of chamber velocity versus distance downstream from injector face for each injector type.

W. T. Olson, USA

3226. **Barrère, M., Graphic representation of various parameters used in the study of rocket motors** (in French), *Rech. aéro.* no. 25, 29–38, Jan.-Feb. 1952.

On the basis of conventional assumptions (adiabatic expansion, complete chemical equilibrium, perfect gas, and one-dimensional flow), the velocity, pressure, and temperature of gases in the expansion section of the nozzle of a rocket motor are calculated. Assumptions made are claimed to hold with good accuracy for low-pressure rockets (chamber pressure between 15 to 30 atmospheres). Elementary dimensionless formulas are derived which are presented graphically. Paper may be useful in a preliminary analysis of performance of rockets for design purposes.

Ahmed D. Kafadar, USA

3227. **Krieger, F. J., Chemical kinetics and rocket nozzle design,** *J. Amer. Rocket Soc.* **21,** 6, 179–185, Nov. 1951.

For hydrogen gas, the effect of chemical kinetics on rocket nozzle design is investigated. Author states that, in evaluating specific impulse on a propellant system, it is standard practice to use either zero or infinite reaction rate. Inclusion of chemical kinetics in such evaluation is important. He also states that specific impulse calculations are made without regard to nozzle configuration and with the assumption of zero entrance velocity of gases from combustion chamber to nozzle. Further, usually conical convergent section is used from combustion chamber to throat. Author uses spherical transition in paper, stating that "The use of a spherical section instead of a convergent cone insures continuity of flow from the combustion chamber into the nozzle entrance." This statement is not explained, and it infers that no continuity of flow exists when conical convergent section is used, which is erroneous.

Thermodynamic, one-dimensional flow, and chemical kinetic equations are written; also, the geometry of the convergent section, the throat, and the nozzle are discussed at length. The thermodynamic data used in the paper are presented. The gas constant as well as the forward reaction rate are taken as constant over the entire range of temperatures and pressures. Next, the step-by-step method of computations is discussed in detail. Composition, pressure, specific impulse, and nozzle length versus temperature are plotted for the three cases of zero reaction rate, infinite reaction rate, and actual reaction rate. Also, for the three cases, the nozzle shape is plotted. The results of the analysis are compared with those made with zero and infinite reaction rates. It is unfortunate that, although it is stated in several places that the expansion process is assumed to be isentropic, in other places of the paper the same process is referred to as adiabatic. Also, the choice of the constant value of the reaction rate constant seems to be unfortunate, since it closely approximates the case of the infinite reaction rate.

It is reviewer's opinion that the study of nonsteady-flow phenomena will give the answer to rocket design. Also, three-dimensional treatment would be necessary, since the phenomena are of such nature.

Paul Torda, USA

3228. **Huff, V. N., Gordon, S., and Morrell, Virginia E., General method and thermodynamic tables for computation of equilibrium composition and temperature of chemical reactions,** *NACA Rep.* 1037, 57 pp., 1951.

Supersedes paper reviewed in AMR **4,** Rev. 1345.

3229. Egerváry, E., and Turán, P., On a certain point of the kinetic theory of gases, *Studia Math.* **12,** 170–180, 1951.

Let E be a cube whose walls are $x = 0$, $x = \pi$, $y = 0$, $y = \pi$, $z = 0$, $z = \pi$. In this cube one assumes there are n particles (n large). The particles are assumed to be dimensionless of equal mass, the impacts on the walls follow the laws of elastic impact, and it is assumed there are no attractive or repulsive forces between the particles and that there are no exterior forces acting on the particles. Assume, further, that two particles never collide. At time $t = 0$, the coordinates $x_{\nu,0}$, $y_{\nu,0}$, $z_{\nu,0}$, $\nu = 1, 2, \ldots, n$ are arbitrary and the velocities satisfy

$$\dot{x}_{\nu,0} = (n + \nu)^2, \quad \dot{y}_{\nu,0} = (n + \nu)^2\sqrt{2}$$
$$\dot{z}_{\nu,0} = (n + \nu)^2\sqrt{3}, \quad \nu = 1, 2, \ldots, n \qquad [1]$$

Let K be any parallelepiped of volume V_k contained in E. Denote by $N(t_0, K)$ the number of particles in K at time $t = t_0$. Authors prove that for any $0 \leqq t \leqq n^{1/4}$, except time intervals whose total length does not exceed $cn^{-1/10} (\log n)^2$ (c an absolute constant), we have for every K

$$\left| \frac{N(t_0, K}{n} - \frac{V_k)}{\pi^3} \right| < n^{-1/10}$$

The theorem shows that, whatever the initial position of the particles, if the velocities satisfy [1] "most of the time" there will be equidistribution. Authors obtain the same result for various other assumptions on the velocities, e.g., $\dot{x}_{\nu,0} = (n + \nu)^{2/5}$, $\dot{y}_{\nu,0} = (n + \nu)^{2/5}\sqrt{2}$, $\dot{z}_{\nu,0} = (n + \nu)^{2/5}\sqrt{3}$. They also obtain the same result if they permit elastic collision between two particles. Also various interesting open questions are discussed.

Courtesy of Mathematical Reviews P. Erdös, USA

3230. Lunbeck, R. J., and Boerboom, A. J. H., On the second virial coefficient of gas mixtures, *Physica* **17,** 1, 76–80, Jan. 1951.

General knowledge about virial coefficients is reviewed briefly. Experimental evaluation of coefficients for any mixture requires an extended set of very accurate measurements. Empirical rules for estimation of coefficients for mixtures are repeated. Agreement between empirical predictions and the small amount of published data is qualitative but not exact.

Jack D. Bush, USA

Heat and Mass Transfer

(See also Rev. 3053)

3231. Datsev, A. B., On the heat conduction of a nonhomogeneous bar (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **82,** 6, 861–864, Feb. 1952.

Consider a thin nonhomogeneous bar of length l which extends along the x-axis from $x = x_0$ to $x = x'$ with a temperature function $u(x, t)$ which satisfies the equation

$$\frac{\partial}{\partial x}\left(k \frac{\partial u}{\partial x}\right) = \rho\sigma \frac{\partial u}{\partial t} \qquad [1]$$

where k, ρ and σ are continuous functions of x. The initial and end conditions are

$$u(x, 0) = \Phi(x), \qquad (x_0 < x < x') \qquad [2]$$
$$u(x_0, t) = \phi(t), \quad u(x', t) = \psi(t), \qquad (t > 0) \qquad [3]$$

where Φ, ϕ, and ψ are arbitrary bounded integrable functions. In this paper, the solution to the above problem is found formally by reducing it to a multilayer problem in heat conduction. The interval (x_0, x') is broken into n arbitrarily chosen parts. In the typical subinterval, the function $k(x)$ is replaced by $k_i = k(x_i)$. The same is done for ρ and σ. Thus for each of the subintervals, equation [1] is replaced by

$$\frac{a_i^2 \partial^2 u_{in}}{\partial x^2} = \frac{\partial u_{in}}{\partial t}, \; a_i^2 = \frac{h_i}{\rho_i \sigma_i}, \; i = 1, 2, \ldots, n \qquad [4]$$

where $u_{in}(x, t)$ is the temperature function in the interval $\Delta x_i = x_i - x_{i-1}$. At the point $x = x_i$, u_{in} satisfy the conditions

$$u_{in} = u_{i+1,n}, \quad k_i \frac{\partial u_{in}}{\partial x} = k_{i+1} \frac{\partial u_{i+1,n}}{\partial x} \qquad [5]$$

The initial and end conditions become

$$u_{in}(x, 0) = \Phi_i(x) = \Phi(x), \; (x_{i-1} < x < x_i) \qquad [6]$$
$$u_{1n}(x_0, t) = \phi(t), \; u_{nn}(x', t) = \psi(t), \; t > 0 \qquad [7]$$

It is shown that $u_{in}(x, t)$ has $u(x, t)$ as its limit as $n \to \infty (x_0 < x < x', t > 0)$ and satisfies [1], [2], and [3].

Courtesy of Mathematical Reviews C. G. Maple, USA

3232. Eckert, E. R. G., and Soehngen, E., Distribution of heat-transfer coefficients around circular cylinders in crossflow at Reynolds numbers from 20 to 500, *Trans. ASME* **74,** 3, 343–346, Apr. 1952.

Forced convection, local heat-transfer coefficients are obtained around the periphery of horizontal cylinders in air for low Reynolds numbers and small temperature differences between the cylinder surfaces and the air. The local coefficients were obtained from consideration of the density fields around the heated cylinders, as delineated by means of the Zehnder-Mach interferometer.

Results show that the point of flow separation is farther downstream from the stagnation point (135°) for low Reynolds numbers than for high Reynolds numbers. The contribution of the ensuing small wake region to the total heat transfer from the cylinder is less than 15%, the predominant heat transfer occurring on the forward portions of the cylinders.

Robert M. Drake, Jr., USA

3233. Ostrach, S., An analysis of laminar free-convection flow and heat transfer about a flat plate parallel to the direction of the generating body force, *NACA TN* 2635, 47 pp., Feb. 1952.

An analysis is made of the free-convection flow about a flat plate oriented in a direction parallel to that of the generating body force (centrifugal force) under the prime assumption that the relative temperature difference is small. It is found that the N_{Gr} is the principal factor determining the type of flow and that for large N_{Gr} the flow is of the boundary-layer type.

Velocity and temperature profiles for N_{Pr} from 0.01 to 1000 were computed on the basis of constant body force and plate temperature, and agreement with experiments with air is seen to be good.

A flow and a heat-transfer parameter are derived as functions of N_{Pr}. Quantities such as shear stress and heat-transfer rate can be computed from these parameters. Comparison of the heat-transfer parameters obtained from experiments, an approximate theory, and the present studies agreed well over a range of N_{Pr} from 0.01 to 1000. It is shown that the commonly used semi-empirical relation for the heat-transfer coefficient will yield good results in restricted N_{Pr} ranges.

Y. S. Touloukian, USA

3234. Pignedoli, A., On a diffusion problem in mathematical physics (in Italian), *Ann. Mat. pura appl.* (4) **32,** 281–293, 1951.

Let D, c, h, Λ be constants, $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$, S a region in space with boundary σ on which n denotes the exterior normal. While author starts out with the boundary-value problem of finding a solution $\rho = \rho(x, y, z, t)$ of $D\Delta\rho - \rho_t + c\rho = q(x, y, z, t)$ in S and $\partial\rho/\partial n + h\rho = 0$ on σ, his assumptions on ρ and q are such as to reduce the problem to finding a solution $u = u(x, y, z)$ of $\Delta u + \Lambda u = f(x, y, z)$ in S and $\partial u/\partial n + hu = 0$ on σ. The procedure for replacing this problem by an integral equation, through introducing a Green function, is sketched. For the special case, S is a sphere, and the given function f can be expanded in terms of surface spherical harmonics, the above boundary-value problem is solved by the method of separation of variables.

Courtesy of Mathematical Reviews F. G. Dressel, USA

3235. Businger, J. A., Heat transfer in the transition zone of a round duct at forced laminar flow (in Dutch), *Ingenieur* **64,** 4, 0.17–0.19, Jan. 1952.

Author calculates an expression for the Nusselt number Nu in the transition zone of a round duct at forced laminar flow. The transition zone is that region in which the velocity distribution of the flowing medium passes from being uniform (entry of the tube) to the Poiseuille distribution. Author deals only with the special case of constant wall temperature.

His calculations are based on Schiller's analytical treatment (growing parabolic sections with increasing distance x in the tube, and a region of uniform velocity u). As Schiller's expression gives rise to a very complicated differential equation for the heat transfer, author approximates Schiller's formula by

$$4x/dRe = 0.11\,(u/\bar{u} - 1)^2$$

d is diameter of the tube, Re Reynold's number, $\bar{u}$ average velocity in a cross section. From this expression, a formula for the velocity distribution in the parabolic sections can be found.

Then author remarks a similarity between the differential equations for heat and flow; for this sake, the term with grad p is omitted from the latter. In this way, author finally obtains an expression for the Nu number.

Reviewer's criticisms, apart from a few trivial misprints, are: (a) Author's approximation for $4x/dRe$ is only rather good in the region: $10^{-2} < 4x/dRe < 10^{-1}$; large deviations from Schiller's formula occur especially in the region $4{\cdot}10^{-4} < 4x/dRe < 4{\cdot}10^{-3}$ (up to 25%), for which region author's expression for the Nu number is calculated. (b) Unfortunately, Schiller's theory seems to give good results only for $x/dRe < 2{\cdot}10^{-2}$. (Measurements by Nikuradse, see e.g., Prandtl-Tietjens, or Schiller's paper in: "Handbuch der Experimental-Physik," or Schlichting: "Grenzschichttheorie.") Author's approximation, however, is only good for $x/dRe > 10^{-2}$. (c) An extra contribution to heat transmission in the transition zone arises from the radial component of velocity, which is necessarily not zero in this zone. This contribution is not taken into account in author's paper; this apart from the omission of the term with grad p, which is likely to be doubtful.

Author informed reviewer that his paper should only be considered as a possible way of calculating heat transmission in the transition zone. For the greater part, he agrees with criticisms mentioned. From this viewpoint, author's paper certainly has value. L. Jansen, Switzerland

3236. Florin, F., Calculation of latent heats of evaporation (in German), *Forsch. Geb. Ing.-Wes.* (*B*) **18,** 2, 36–44, 1952.

The Clausius-Clapeyron equation is transformed by use of a previously determined empirical "temperature function" which was derived from observed data on the relation between vapor pressure and temperature. The resulting expression facilitates the computation of heats of evaporation. Furthermore, an independent check is obtained for the "temperature function"; Trouton's rule is generalized and corrected; simple expressions are derived for the two empirical constants appearing in Thiesen's relation between heat of evaporation, critical temperature, and temperature; a useful correlation is presented between the molar heat of evaporation of normal paraffins and the corresponding temperatures and vapor pressures.

From author's summary by S. S. Penner, USA

3237. Gregorig, R., and Tommer, H., Reduction of heat transmission due to nonuniform velocity distribution and inaccurate tube pitch (in German), *Schweiz. Bauztg.* **70,** 11, 12; 151–155, 174–176; Mar. 1952.

The results of a mathematical investigation made under simplified assumptions have been confirmed experimentally on a simple cross-flow heat exchanger consisting of 2×5 tubes of 18-mm diam. It has been shown that the influence of (linear) change of velocity across the oncoming flow is practically negligible. For the velocity ratio $w_{\max}/w_{\min} = 10$ and $kF/Gc_p = 2$, the reduction of heat transfer is about 2%. As a consequence of change in tube pitch (23 + 28 instead 2 × 28 mm), a reduction of transfer coefficient of 3–4% has been found. If two adjacent rows of tubes are in contact, the decrease is 32%. Mathematical methods gave somewhat higher values. With a flow parallel to the tubes, the influence of pitch irregularity was substantially less.

Otakar Maštovský, Czechoslovakia

3238. Green, L., Jr., Gas cooling of a porous heat source, *J. appl. Mech.* **19,** 2, 173–178, June 1952.

See AMR **5,** Rev. 1882.

Acoustics

3239. Mayo, C. G., Standing wave patterns in studio acoustics, *Acustica* **2,** 2, 49–64, 1952.

The sound response of a rectangular room to a point source of impulsive sound consists of a series of pulses, reflected by the walls. The response to a continuous tone started abruptly is built up as the vectorial sum of the reflections. Three stages are distinguished: (1) Echoes are few and large; (2) echoes are numerous and smaller, the images of the source lying on straight lines give rise to frequencies, which are not eigentones; these decay inversely as time, whether or not there is absorption; (3) echoes arrive nearly in synchronism from images lying in planes and establish eigentone pattern proper; the decay is exponential. The general exposition is followed by a mathematical treatment. Reviewer is not sure of the validity of extrapolating results to nonrectangular rooms. R. Vermeulen, Holland

3240. Velizhanina, K. A., Experimental investigation of resonant sound absorbers (in Russian), *Zh. tekh. Fiz.* **21,** 9, 1087–1099, Sept. 1951.

Reference is made to work of Rzhevkin who showed the possibility of obtaining a high coefficient of absorption by using a system of resonators regularly spaced on the surface of a wall, as, e.g., a sheet of metal or wood placed at some distance from the wall and perforated with holes containing porous material. Formulas are derived for obtaining the parameters of such a resonant absorber with rigid front wall which is required to have an absorption coefficient greater than a given value of some fundamental frequency, but it is pointed out that the calculations are not satis-

factory if the depth of the resonator cavity is greater than $^1/_4$ the corresponding wave length. Under practical conditions, the front wall of a resonant absorber may be excited to vibration by the incident sound waves, and a theoretical method, confirmed by experiment, is developed for calculating the influence of this effect on the frequency characteristic of the absorption coefficient. Experimental results show that, where the frequency of the fundamental mode of vibration of the wall is near to or equal to the frequency of the resonator cavity, there occurs in the region of the maximum in the frequency characteristic of the absorption coefficient a dip which considerably distorts the characteristic of the resonant absorber, though by increasing the damping of the vibration of the wall the dip can be smoothed out. If the natural frequency of the wall is considerably higher than that of the resonator cavity, then the resonant absorber behaves as though the wall were absolutely rigid and the influence of the vibration of the wall in the region of the cavity-resonance frequency cannot be taken into account. The effect of the thermal conductivity of the wall on the dissipative loss and, hence, on the absorption coefficient is also discussed.

The resonant absorbers treated in this work do not contain porous material, and the dissipative loss is achieved simply by friction or the orifices of the resonators themselves. It is suggested that, by using this new principle, resonant absorbers can be constructed of perforated sheets with a relatively small number of holes per unit area; and since such a structure is very easily made it should prove of practical significance.

Marie Goyer, England

3241. Borgnis, F. E., On the theory of the fixed path acoustic interferometer, *J. acoust. Soc. Amer.* **24,** 1, 19–21, Jan. 1952.

A general expression is given for the electric input impedance of the acoustic interferometer. From this expression, formulas are derived for determining the velocity of sound by varying the frequency, or for determining changes in velocity due to variations of pressure, temperature, etc. In papers dealing with the fixed-path interferometer, one commonly finds the suggestion that the actual path length needs some correction when the path ends at a nonperfect reflector. According to author, no such correction is indicated by the theory. From author's summary

3242. Somerville, T., and Gilford, G. L. S., Composite cathode ray oscillograph displays of acoustic phenomena and their interpretation, *B.B.C. Quart.* **7,** 1, 41–53, 1952.

The logarithmic display of the reverberation following a short pulse of warbled tone is produced on a cathode-ray tube by means of apparatus described at length in *Electronic Eng.* **23,** 283, 284, 285, 286, Sept.-Dec. 1951. "Pulsed glide displays" consist of successive logarithmic decays at progressively increasing frequencies, photographed on a film moving at a speed of 1 mm/sec. The families of displays make up patterns characteristic of the over-all behavior of the studio and give a coded pictorial representation of the individual behavior in specific frequency domains. The interpretation is discussed, though still at an elementary stage. Typical examples are given. The method has proved of great use as a diagnostic aid and as an adjunct to careful listening to trace defects and prescribe remedies.

R. Vermeulen, Holland

3243. Davenport, W. B., Jr., Johnson, R. A., and Middleton, D., Statistical errors in measurement on random time functions, *J. appl. Phys.* **23,** 4, 377–388, Apr. 1952.

Experimental time averages of properties of random time functions depend upon the averaging interval as well as the function. Paper shows how these finite time-average estimates may differ from the true value. Application is made to low pass filters, spectrum analyzers, and correlators.

Paper refers to an apparently new and unique heterodyne system for determining power frequency spectra. Reviewer recalls two such devices for this purpose on the market for over ten years.

Thomas Caywood, USA

3244. Rzhevkin, S. N., The connection of the problem of sound diffraction by a sphere with the reciprocity theorem (in Russian), *Zh. tekh. Fiz.* **21,** 10, 1224–1227, Oct. 1951.

Point A is on the surface of a fixed rigid sphere, point B at a large distance from the sphere. Author verifies a reciprocity result for the pressure produced at A or B by point sources at B or A; this he does by direct solution of the boundary problems, subject to approximations because of the large distance.

Courtesy of Mathematical Reviews F. V. Atkinson, Nigeria

3245. Richards, D. L., Design and analysis of subjective acoustical experiments which involve a quantal response, *Acustica* **2,** 2, 83–91, 1952.

The final judgment of acoustic quality can only be obtained by subjective measurements. These are usually concerned with relating the subjective response to some physically measurable stimulus. Only very simple classification judgments or quantal responses are possible. The methodology of biological experimentation is also appropriate in this field. Examples are given of application of "probit analysis" and "analysis of variance." These statistical methods not only yield valid confidence intervals but also point to efficient methods of conducting the measurements through separation of sources of variation.

R. Vermeulen, Holland

3246. Meyer, E., and Karmann, R. W., Vibration of air particles in the neighborhood of a sound-absorbing wall (in German), *Acustica* **1,** 3, 130–136, 1951.

The sound field near an absorbent wall of known impedance was investigated photographically in the range 100–500 cps with the aid of talcum particles. The particles were suspended in a wave guide of square cross section, part of whose lower wall consisted of either a glass wool slab or resonators with absorbent cotton. In general, the particles follow elliptical trajectories; for a nonabsorbing wall, they reduce to straight lines parallel to the surface. The ratio of the minor and major axis, and the angle of the second with the surface, were measured for various ellipses; these are in good agreement with theoretical values deduced from Morse's expression for the ratio of the velocity components perpendicular and parallel to the surface. Experimental and theoretical plots of these quantities as functions of the frequency are presented, as well as are several fascinating photographs of the particle trajectories. Aside from its academic significance, the work presents an interesting measuring technique and is of value in obtaining an intuitive understanding of the behavior of sound-absorbing surfaces. V. Twersky, USA

3247. Nomura, Y., and Aida, Y., On the radiation impedance of a rectangular plate with an infinitely large fixed baffle, *Sci. Rep. Res. Inst. Tôhoku Univ.* (*B*) **1, 2,** 3, 337–347, Mar. 1951.

Formulas for calculating the radiation impedance of the rectangular plate which vibrates normally to the plane of an infinitely large fixed baffle are obtained from the pressure distribution on the plate. The energy distribution at a large distance from the vibrating plate has been calculated by the Stenzel method, and the radiation resistance may be calculated from it, but not the reactance component. In this paper, authors give the general solution of the problem and some numerical discussions. The

mathematical treatment is correct. The results of calculation of $R/\rho c$ and $X/\rho c$ for various values of k are shown in two figures. For comparison, curves of a circular plate and an infinitely long ribbon are plotted. It appears that the square plate (for which $k = 1$) approaches the circular plate most closely, and, as k becomes greater, the curves approach more and more closely those of the infinitely long ribbon. With regard to rectangular plates, authors claim that the relation for k is complicated and cannot give any conclusion for the case of $R/\rho c$.

S. M. Uzdilek, Turkey

3248. Malecki, I., Sound insulation of floor structures (in Polish), *Inżyn. Budown.* **9**, 2, 43–51, Feb. 1952.

In theoretical part of the paper, author analyzes influence of sound absorption of rooms upon the resultant insulation ability of a partition. To check the theoretical considerations, seven different types of floor structures with nine different kinds of subfloor finish were set up in an experimental building of the Institute of Building Technique in Warsaw, and a considerable number of measurements were made. In all, 413 tests were made, sound-insulation coefficient of structural floor (i.e., without subfloor finish) being measured first and the coefficient of complete floor structure being measured next. In that way, valuable experiments data were acquired and best constructions chosen.

The best results were obtained with hollow-clay tile floors of Ackermann and ZP1 types. Quoting of some of the results may be of interest; they can be used widely in building design.

TABLE No. 1. Sound insulation coefficient of bare structural floors.

Type of structural floor	Frequency in cps						Insulation coefficient for solid-borne sounds in dB
	128	256	512	1024	2048	4096	
DMS	39.4	42.5	48.0	48.0	46.7	77.2	42.0
ZP1	34.8	39.6	50.6	54.0	57.8	63.0	53.6
Ackermann	45.3	50.8	51.7	58.2	68.8	72.3	48.2
Hourdis	35.4	45.6	47.5	49.0	58.4	59.2	47.5
Italian	43.3	48.7	48.9	52.0	58.9	68.3	42.5

TABLE No. 2. Sound insulation coefficient of ZP1 type of structural floor with subfloor finish

Kind of subfloor	Frequency in cps						Insulation coefficient for solid-borne sounds in dB
	128	256	512	1024	2048	4096	
Rock wool 4 cm	53.4	50.5	57.2	63.0	71.7	78.1	60
Glass wool 4 cm	58.6	55.7	57.9	65.1	71.9	75.0	57
Lime plaster 5 cm	51.6	53.7	57.9	65.1	69.9	78.4	60
Peat boards 3 cm	50.4	53.5	59.9	63.0	69.9	74.2	59
Wood-cement boards 1.5 cm	53.6	50.7	59.9	65.1	71.9	75.4	61
Flax plant file 1.5 cm	53.6	44.7	56.9	61.1	67.9	71.4	61
Wood fiber boards 1.5 cm	60.6	51.7	60.9	65.1	72.9	74.4	60
Wood fiber board 1.5 cm weighted by reinforced concrete board 3 cm.	54.4	56.5	60.9	65.0	71.7	73.2	67

The foregoing data show best results are obtained with wood fiber boards weighted by the reinforced concrete slab, the latter preventing resonant vibrations of the whole structure. Results of measurements and theoretical considerations show also that insulating ability of bare hollow-clay tile floor is insufficient, and when used for prefabricated structural floors, covering of the surface with some additional absorbents is necessary.

K. Zarankiewicz, Poland

3249. McGrath, J. W., Note on Beranek's theory of the acoustic impedance of porous materials, *J. acoust. Soc. Amer.* **24**, 3, 305–309, May 1952.

Beranek developed in 1942 a relatively simple and easily applicable theory of the acoustic impedance of porous materials. This paper presents a modification of his theory which avoids two approximations used formerly. It is found that the specific dynamic frictional resistance and the effective density of the material are functions of frequency. The modified theory is applied to two common acoustic materials, and it appears to be quite adequate for materials of large porosity and small flow resistance. It is less adequate for other materials.

Beranek presented a second theory [title source, **19**, 556, 1947] which is more nearly complete and which allows for propagation of sound both in "skeleton" of the material and in interstices.

From author's summary by E. G. Fischer, USA

3250. Lawley, L. E., The propagation of sound through gases contained in narrow tubes, *Proc. phys. Soc. Lond.* (*B*) **65**, part 3, 387B, 181–188, Mar. 1952.

Measurements of velocity and attenuation of sound at 60 to 120 kcps in air, oxygen, nitrogen, and hydrogen contained in tubes of 0.15 to 1.17-mm diam verify the form of the Helmholtz-Kirchhoff equations. Experimental viscosity-conductivity constant is about 5% higher than that calculated from accepted values of viscosity and thermal conductivity.

Martin Greenspan, USA

Ballistics, Detonics (Explosions)

(See also Revs. 3145, 3146, 3227)

3251. Rateau, A., Study of motion of projectiles by "snapshot" photography (in French), *Actes Coll. inter. Mécan.* **II**, *Publ. sci. tech. Min. Air, Paris*, no. 250, 73–80, 1951.

Author describes a method for determining photographically the angle which the axis of a projectile while in flight makes with its trajectory. A mirror arrangement is used to obtain two simultaneous stereoscopic pictures. The duration of the exposure is about one microsecond. J. S. Rinehart, USA

3252. Nishiwaki, J., Resistance to the penetration of a bullet through an aluminum plate, *J. phys. Soc. Japan* **6**, 5, 374–378, Sept./Oct. 1951.

The resistance offered by an aluminum plate to its penetration by a bullet is considered to be the sum of a constant force proportional to the thickness of the plate and the nose angle of a (conical) bullet, and a force resulting from the momentum imparted by the bullet to the aluminum plate and depending upon the bullet speed and the nose angle. The force assumed independent of velocity was obtained as a function of plate thickness by pushing a mandril slowly through the plate. Bullets with varying nose-cone angles were fired through a series of plates, and the velocities before and after perforating the plates were measured. In general, good agreement was obtained between the theoretically predicted velocity loss and that attained experimentally.

Herbert K. Weiss, USA

3253. Fayolle, P., Short-time measurements (in French), *Actes Coll. inter. Mécan.* **II**, *Publ. sci. tech. Min. Air, Paris,* no. 250, 51–64, 1951.

This review of short-time measurements for ballisticians is lavishly illustrated. Of the various detectors, break screens mounted on glass are reliable to within 10^{-6} sec, with photocells and ionization probes better and microphones worse by factors of order 10. The best electromechanical record (Kerr cell with rotating-drum camera) can be read to better than 10^{-7} sec. The best cathode-ray records, television, or spiral types are readable to 10^{-7} sec. Counter chronographs have so far only been taken to 10^{-6}-sec pulse interval. [See Fayolle and Naslin, AMR **2**, 1063.]

H. H. M. Pike, England

3254. Ubbelohde, A. R., Transition from deflagration to detonation: the physico-chemical aspects of stable detonation, Third Symp. Combust. Flame Expl. Phenom.; Baltimore, Md., Williams & Wilkins, 566–571, 1949. $13.50.

3255. Muraour, H., and Aunis, G., Study of laws of combustion of colloidal powders (in French), *Mém. Artill. fr.* **25**, 1, 117–165, 1951.

Paper includes a statement of the authors' position in a controversy now 20 years old and ten papers old, and several added notes on the burning of solid propellants. Records of pressure developed in closed chambers by the burning of propellants are presented as proof of the authors' conclusion that the rate at which a colloidal propellant burns is best expressed as a linear function of the pressure, and that the rate is not, as suggested by Crow and Crimshaw [first in *Phil. Trans. roy. Soc. Lond.* (*A*) **230**, 387–411, 1932], proportional to the density of the gaseous combustion products. Muraour has given a theory in which the rate of burning, limited by the conduction of heat from the gaseous products to the surface of the burning solid propellant, turns out to be proportional to the pressure. Authors discuss additions to this theory which would serve to explain the appearance of the constant term in the observed linear function of pressure. There is pointed out, and an explanation proposed for, a lack of expected progressivity in the burning of seven perforated powders extruded with solvents. There is also a discussion of the phenomenon of erosive burning of long tubes which was first reported by Muraour.

William C. Taylor, USA

Soil Mechanics, Seepage

(See also Revs. 3078, 3089)

3256. Ornatskii, N. V., Soil mechanics [Mekhanica gruntov] (in Russian), Izdat. Moskov. Univ., 1950, 419 pp.

Book is intended as a text for geological students in an orientation course in soil mechanics. In spite of this limited application, the book is wide in scope and, except for the subject of seepage, which is not treated, it covers more topics than are found in recent American texts on soil mechanics. The book is purely theoretical with a strong emphasis on mathematics, while the experimental side is barely mentioned. Numerical examples, designs, and problems are omitted entirely.

Contents consist of introduction, containing the fundamentals of theoretical mechanics, and two main parts; the first is intended to present the basic theories or viewpoints of soil mechanics, and the second, the applications. The first part covers strength of materials of solid body, theory of cohesionless materials, cohesive materials, soil-water combination, soil-water-gas combination, soil as a dispersed system, and some extensions and correlations of these theories. The second part contains stresses in the ground on the basis of Fröhlich's stress-concentration theory, one-dimensional consolidation without secondary effects, settlement of foundations, and a number of miscellaneous topics. Separation of the contents into theory and applications is not carried out consistently.

Author leans heavily on Russian sources, and out of some 50 items in the bibliography only five are foreign, none of which is recent. Credit in development of the subject is given to a score of Russian engineers and scientists, but to no foreigners. In preface, it is stated that the presentation is made on the basis of application of dialectic materialism to soil mechanics.

Contents of the book have considerable element of novelty in their approach and emphasis. Their quality is varied. There is an excellent discussion on moisture of adsorption, capillarity, and the stresses brought about by the latter. Instructive is the chapter on soil-water-gas combination and its transformations due to variation in temperature and loading. The theory of soil as a dispersed system is new to reviewer. It presents a method of statistical study of variations of soil characteristics from point to point, described in terms of the average value of the characteristic and of its range of dispersion. Even though the direct utilization of this theory is not apparent, its novelty is interesting.

A number of details may be criticized. Thus, there is a total absence of reference to shear strains, either linear or volumetric, accompanying shear stresses, which results in incompleteness and even misinterpretation in explanation of soil phenomena, particularly in connection with general discussion of shear strength of soil material, shear failure due to sudden liquefaction, and the ribbon method of determining the coefficient of lateral pressure, consisting in comparing the pull-out resistances of vertical and horizontal ribbons, embedded in soil contained in a modified consolidometer. The last example points also to a lack of recognition of the effects of disturbance unavoidable in placement of the ribbons.

The hydrodynamic seepage force is found erroneously $e/1+e$ times what it should be, yet in a later application its correct magnitude is used. A strong leaning on speculative reasoning is evidenced in the problem of the initial stage of two-dimensional consolidation, which is solved on the basis of the assumption of seepage without a change in void ratio, and some other unjustified assumption.

Rather surprising is the emphasis placed on the so-called "principle of hydrocapacity" which is merely an assertion to the effect that the variation in the void ratio in the course of a single period of compression with any degree of lateral expansion is solely determined by the sum of the three principal stresses. The discussion leading to the enunciation of this principle is peculiar in its manner and logic, and cannot be classed as sound reasoning based on recognized principles of mechanics.

Despite several important objections, book is interesting to a western reader for the unfamiliarity of its approach and treatment, reflecting the development of a scientific thought along a separate and somewhat independent path.

Alexander Hrennikoff, Canada

3257. Toth, L., Pore pressure (in French), *Tech. mod. Constr.* **7**, 1, 2; 3–8, 50–56; Jan., Feb. 1952.

Pore-pressure importance in earth-dam construction is analyzed. After short introduction on first developments made at the Bureau of Reclamation since 1936, author defines pore pressure of fluids filling voids of a soil. These are usually air and water. As soil particles and water are considered incompressible, when an embankment is compressed either air is compressed or dissolved by water, or both fluids are expelled from soil. Bureau of Reclamation possesses enough experience to state that pore pres-

sure is, in many cases, greater than hydrostatic pressure of highest waters.

Causes of pore pressure are analyzed. First, author considers consolidation. When embankment is made with soils of low permeability, pore pressures are slowly dissipated by percolation, and consolidation takes place before fluid is evacuated. Second cause of pore pressure is compaction of embankment because sheep's-foot roller develops high pressures that are equally absorbed by entrapped air, and causes subsequent percolation of water. Third origin of pore pressure is the percolation of water from upstream face of dam during normal operation.

Bureau of Reclamation now tends to placing soils with 3 to 5% lower moisture than optimun Proctor value, and then utilizing increased weight rollers.

Methods of estimating pore pressure are analyzed, including theoretical formulas based on Boyle and Henry laws. Finally, USBR apparatus to measure pore pressure in earth dams is described. A. Balloffet, Argentina

3258. **Taylor, D. W., A triaxial shear investigation on a partially saturated soil,** "Triaxial testing soils bitum. mixtures," *ASTM Spec. tech. Publ.* no. 106, 180–187, 1951. $3.50.

A method is described for determining the shear strength characteristics of a partially saturated silty clay. Multiple-stage triaxial tests are used for this purpose. In such tests a specimen is loaded axially to failure under a given confining pressure and then, after being subjected to an increased confining pressure, it is again loaded to failure. This process is repeated for several stages of loading.

Multiple-stage tests are applicable to soils that are not sensitive to changes of structure and, for such soils, this type of test gives as much information concerning shear strength characteristics as can be obtained from several conventional tests. Furthermore, since only one test specimen is required, errors resulting from variations in test specimens are eliminated.

A procedure is outlined for determining the shear strength for a given soil deposit with consideration given to the effects of variations in overburden pressure, preconsolidation pressure, and ultimate pressure. The procedure makes allowance for increased strength resulting from compression of a partially saturated soil when no water-content changes occur. Consistent results are obtained. R. E. Fadum, USA

3259. **Schwartz, B., Fundamental study of clay: XII, A note on the effect of the surface tension of water on the plasticity of clay,** *J. Amer. ceram. Soc.* **35**, 2, 41–43, Feb. 1952.

The effect of lowering the surface tension of water on the plasticity of a clay was evaluated from stress-strain diagrams. Lowering the surface tension decreased the yield point, maximum strength, and workability of the plastic clay. A theoretical explanation for these phenomena is presented.

From author's summary by R. E. Fadum, USA

3260. **Muhs, H., Investigation of settlement behavior of foundation soil** (in German), *Bautechnik* **29**, 2, 25–30, Feb. 1952.

In 1941 in Berlin, a loading test on a very large scale was executed. A very stiff foundation raft of 100 m², at a depth of 18.20 m below soil surface, was loaded with 12,500 tons. Below the depth of foundation the subsoil consisted of a stratum of glacial loam of 5.20 m underlain by coarse sand. For the measurement of the settlements of the subsoil, reference points were placed in the foundation itself and at a depth of 2 and 4 meters below it. In comparing the observed settlements with the results of settlement computations, different concentration factors were introduced into the stress-distribution formulas. It was found that the theory of Jelinek, which gives decreasing foundation stresses in increasing foundation depths, could not be applied. The best suitable concentration factor proved to be $\nu = 3$. Reviewer considers it regrettable that no reference point has been placed on top of the sand layer at 5.20 m below the foundation depth, which should have enabled the writers to measure the settlements of the sand directly. F. C. de Nie, Holland

3261. **Dubose, L. A., Evaluating Taylor Marl clay for improved use in subgrades,** *Texas Engng. Exp. Sta. Res. Rep.* 35, 17 pp., Mar. 1952.

Report presents results of laboratory soils tests on Taylor Marl, a swelling clay soil associated with poor highway-pavement behavior. Tests performed were classification, compaction at various compactive efforts, unconfined compression, triaxial shear, and special swell tests. Procedure was developed to predetermine the compactive effort necessary to produce a given maximum density at a specific optimum moisture content. Swell tests developed relationships between moisture, density, loading, and swell or consolidation. It was concluded that, to prevent swell in highway subgrades, the soil should be placed at a low density and high moisture content. Shear data indicate a very low strength for these conditions, which reviewer believes is not adequately discussed in terms of highway-pavement design. Only the briefest mention is made of the difficulties which may be encountered in placing the soils at low densities and high moisture contents in the field. Woodland G. Shockley, USA

3262. **Barenblatt, G. I., On some unsteady motions of a liquid and gas in a porous medium** (in Russian), *Prikl. Mat. Mekh.* **16**, 1, 67–78, Jan./Feb. 1952.

This paper gives exact solutions of differential equations which arise in the theory of moving gas under various boundary conditions which, according to the author, are of practical interest.
Courtesy of Mathematical Reviews H. P. Thielman, USA

Micromeritics

3263. **Collins, R. E., Determination of the transverse permeabilities of large core samples from petroleum reservoirs,** *J. appl. Phys.* **23**, 6, 681–684, June 1952.

As a theoretical basis for the determination of the transverse permeability of a large cylindrical core, the flow of a gas in a direction perpendicular to the axis of the cylinder is calculated. The same problem is studied by means of an electrolytic model.
H. C. Brinkman, Indonesia

Geophysics, Meteorology, Oceanography

(See also Rev. 3213)

3264. **McVittie, G. C., Theory of development and of thickness patterns,** *Tellus* **4**, 1, 8–20, Feb. 1952.

Inherent mathematical approximations of Sutcliffe's theory are displayed and shown to be difficult to interpret physically.
J. C. Freeman, USA

3265. **Perkins, D. T., The response of balloons to the wind,** *Bull. Amer. meteor. Soc.* **33**, 4, 135–139, Apr. 1952.

Winds aloft are normally measured by releasing a free balloon and following its motion. Underlying this procedure is the assumption that the horizontal motion of such a balloon is the same as the horizontal motion of the surrounding air. This as-

sumption is not strictly correct since the balloon must pass vertically through a wind field that is not constant with height and, therefore, its motion must exhibit a lag effect.

It can be easily computed from the general solution of the differential equation of horizontal motion of a balloon that, very nearly, $h = 3/\alpha_0$ ($\alpha_0 = C_D\rho A/2M$, where C_D is a coefficient depending on the Reynolds number and the shape of the balloon, ρ is the density of the air, A is the area of cross section of the balloon, h is the thickness of the layer below the height of the balloon which encloses 95% of the integrated wind effect on the measured wind, and M is the mass of the balloon). Author presents other transformations of this result, introducing the value of the lift ratio.

L. J. Tison, Belgium

3266. Kuo, H.-L., Dynamical aspects of the general circulation and the stability of zonal flow, *Tellus* **3,** 4, 268–284, Nov. 1951.

Author utilizes the linearized vorticity equation with an introduced harmonic perturbation to determine stability of certain profiles in atmospheric motions. Assumptions of nondivergence and conservation of kinetic energy limit applicability to motions in upper troposphere.

Zonal profiles in which the perturbations are amplified or damped are discussed mathematically, and the case of the neutral disturbance is investigated numerically. Atmospheric profiles in which disturbances grow or die out certainly exist, and author points out that the amplified disturbance which withdraws energy from basic current acts as a brake to the general circulation, whereas the damped disturbance supplies energy to the zonal flow.

Warren W. Berning, USA

3267. Ogura, Y., The theory of turbulent diffusion in the atmosphere, II, *J. meteor. Soc. Japan* **30,** 2, 53–58, Feb. 1952.

Article extends author's theory of turbulent diffusion to the case in which the time of observation of the phenomenon is finite. Here only those eddies smaller than a certain scale (which is related to the time of observation) are effective for the turbulent diffusion process, as opposed to the case of infinite time of observation previously treated, where all eddies were effective.

Lester Machta, USA

3268. Bowden, K. F., and Fairbairn, L. A., Further observations of the turbulent fluctuations in a tidal current, *Phil. Trans. roy. Soc. Lond.* (*A*) **244,** 883, 335–356, Mar. 1952.

The turbulence in tidal currents near the bottom of Mersey estuary has been recorded by means of two Doodson current meters. Within the range of periods covered (2 seconds to 2 minutes), the power spectrum is peaked at the short periods and falls off for longer periods without showing any predominant period bands (except those identified with surface waves). In water of 14-m depth the integral scale of turbulence is seven meters in the direction of mean flow, and perhaps one third as large in the vertical and lateral directions.

Walter H. Munk, USA

3269. Båth, M., Earthquake magnitude determination from the vertical component of surface waves, *Trans. Amer. geophys. Un.* **33,** 1, 81–90, Feb. 1952.

Work of Gutenberg and Richter on earthquake magnitude scales is extended to case of vertical component records of surface waves of periods 17–23 sec. An empirical magnitude formula is developed, based on Pasadena records of 305 earthquakes with shallow and intermediate focal depths; author states that the magnitude thus derived from records at one station is uncertain by at least $\pm$ 1/4. Ratio of vertical and horizontal amplitudes of surface waves is found to vary significantly with the magnitude.

K. E. Bullen, Australia

3270. Kaplan, L. D., On the pressure dependence of radiative heat transfer in the atmosphere, *J. Meteor.* **9,** 1, 1–12, Feb. 1952.

Recent experimental evidence seems to establish the validity of Lorentz (linear) pressure broadening of spectral lines under atmospheric conditions. This simplifies the analytical representation of the fractional transmission by atmospheric layers in which the pressure decreases with height.

A transmission function is derived and tabulated, with use of the Elsasser assumption of lines of equal intensity and equal spacing. It is found that further approximations made in the construction of the Elsasser diagram result in overestimation of cooling rates with its use.

Stratospheric heating and cooling rates caused by carbon dioxide are calculated for the NACA Standard Atmosphere. Considerable radiative-flux divergence is found in the 15-μ band.

Neglect of the pressure effect has resulted in underestimation of outgoing radiation by previous investigators of the heat balance. Thus a smaller value of the albedo is required, in agreement with Fritz's findings.

From author's summary by Robert O. Reid, USA

3271. Goody, R. M., A statistical model for water-vapour absorption, *Quart. J. roy. meteor. Soc.* **78,** 336, 165–169, Apr. 1952.

The dynamics of the motion at an idealized frontal surface are considered theoretically. For this purpose, the front is treated as a simple discontinuity of density, but it is assumed that horizontal and vertical gradients of temperature and wind may exist within the air masses. It is shown that at a front of this character between two nonhomogeneous air masses, there must be a discontinuity of vertical velocity at the frontal surface, i.e., there must be up-sliding or down-sliding. The discontinuity in vertical velocity is related to the gradients of wind in the two air masses and is shown to be of a similar magnitude (or only slightly smaller) than the vertical velocities believed to occur in frontal regions.

From author's summary by Robert J. Mindak, USA

3272. Van Mieghem, J., Balance of the absolute vorticity in the atmosphere (in French), *Tellus* **3,** 4, 297–300, Nov. 1951.

The author derives a differential equation for the rate of change of absolute vorticity as apparent to an observer in an arbitrary reference frame, with respect to which the fluid velocity is v. The result, $\partial\xi/\partial t + \text{div}\,(\xi v - v\xi) = \text{curl}\,a$, where a is the acceleration, might have been obtained by purely kinematic means [cf. G. Jaffé, *Phys. Z.* **22,** 180–183, 1921; the reviewer, *Phys. Rev.* (2) **73,** 510–512, 1948]. The author calls the term ξv "convective flux," the term $v\xi$ "nonconvective flux" [cf. the reviewer, loc. cit. and *C. R. Acad. Sci. Paris* **227,** 757–759, 821–823, 1948]. He uses the results to obtain and interpret equations governing the rate of change of a meteorological observable. A similar device was used by the reviewer to derive the Bjerknes theorem [Three lectures on mathematics and mechanics, *Nav. Res. Lab. Theor. Mech. Sect., Mem.* 3836–1, pp. 21–37, 1949] and the Ertel theorem [AMR **4,** Rev. 4228; cf. also Ertel and Köhler, AMR **3,** Rev. 1318].

C. Truesdell, USA

3273. Syōno, S., On the structure of atmospheric vortices, *J. Meteor.* **8,** 2, 103–110, Apr. 1950.

Several features of atmospheric vortexes (symmetric tropical cyclones) are examined kinematically by the vorticity and circu-

lation theorems. First, generation of a region of negative relative vorticity around a tropical cyclone is derived from the vorticity theorem, and then its necessity is shown from the divergence of integrals of the kinetic energy and the pressure difference. From the data of the Okinawa typhoon of 1924, the existence of this region of negative vorticity is verified. Secondly, the circulation theorem is used to examine the effect of latitude on wind. Thirdly, generation of the storm eye is explained from a dynamical point of view. Finally, an application of the vorticity theorem is given.
From author's summary

3274. Yih, C.-S., On a differential equation of atmospheric diffusion, *Trans. Amer. geophys. Un.* **33,** 1, 8–12, Feb. 1952.

With the assumptions that the velocity over a surface varies as the distance y from the surface raised to the power m; that the exchange coefficient varies as the distance y from the surface raised to the power n; that the velocity and exchange coefficient at the outer edge of the boundary layer are independent of x, the distance along the surface; and that the thickness of the boundary layer is independent of x—the equation of diffusion becomes

$$y^m \frac{\partial c}{\partial x} = D \frac{\partial}{\partial y}\left(y^n \frac{\partial c}{\partial y}\right)$$

where c is the vapor concentration at any point, D is a constant, and m and n are constants and independent of one another. Exact closed solutions of this equation are obtained for: (1) Diffusion from a line source embedded in a smooth surface; (2) diffusion from a smooth surface; (3) vapor concentration in the wake of an evaporating surface. Solutions (1) and (2) are similarity solutions and are found with the aid of dimensional analyses. Solution (3) is obtained from solutions (1) and (2). Diffusion in Couette flow is given as an example of solution (3).
Neal Tetervin, USA

3275. Kanai, K., On the M_2-waves (Sezawa-waves), *Bull. Earthq. Res. Inst., Tokyo Univ.* **29,** 1, 39–48, Mar. 1951.

Author discusses the range of the existence, dispersive characteristics, and displacements of a second type of Rayleigh wave (Sezawa wave) in a two-layered elastic half space.
Charles B. Officer, Jr., USA

Lubrication; Bearings; Wear

3276. Blok, H., War on wear, *Engineering* **173,** 4502, 4503; pp. 594, 625–626, May 1952.

Wear is the undesirable migration of the material of a solid surface due to overstressing caused by mechanical forces through continuous or repetitive motion relative to a fluid or solid in contact. Wear may be single-sided, as in the cavitation erosion by a fluid, or double-sided for two mating solid surfaces. In the latter case, besides the temperature effects, the surface contact may be dispersed or concentrated, the surface finish may present summits to plastic flow, and the surface pressure may exist with or without sliding. Pitting of gear teeth is a surface fatigue which is often arrested by strain-hardening, a self-restoring factor. Abrasive wear may be caused by an interference or cutting action, or by the interlocking of the surface irregularities. The wear particles cause further erosive abrasion. Adhesive wear occurs between materials having high adhesive qualities—junctions may be formed requiring the forced separation by tearing and shearing.

Protective measures to reduce wear are discussed under three classifications: (1) The principle of the protective layer includes protective plating and chemical coatings, and contact inhibitors such as hydrodynamic fluid films, boundary layers, and the absorption properties of extreme pressure lubricants; (2) the principle of conversion considers the use of lubricants, surface materials, and proper design to convert destructive to permissive wear; and (3) the principle of diversion proposes the diversion of wear from one surface to another.

Wear is controlled by the choice of lubricants, materials, and design. Further research is suggested particularly for quantitative analysis in the problem of wear. The paper is comprehensive; these notes are in outline form.
J. J. Ryan, USA

3277. Macks, E. F., Anderson, W. J., and Nemeth, Z. N., Influence of lubricant viscosity on operating temperatures of 75-millimeter-bore cylindrical-roller bearing at high speeds, *NACA TN* 2636, 47 pp., Feb. 1952.

A 75-mm-bore (size 215) cylindrical-roller inner-race-riding cage-type bearing was used in an experimental investigation of the effect of oil viscosity on bearing operating characteristics over a range of DN values (bearing bore in mm times shaft speed in rpm) from 0.3×10^6 to 1.2×10^6. Kinematic viscosity at the inlet temperatures varied from 1.7 to 390 centistokes (absolute viscosities of 2.0×10^{-7} to 470×10^{-7} reyns).

A previously developed cooling-correlation analysis was extended to include fully the effect of varying lubricant viscosity. This improved correlation makes it possible to predict either the inner- or the outer-race bearing temperatures from single curves regardless of whether speed, load, oil flow, oil-inlet temperature, oil-inlet viscosity, oil-jet diameter, or any combination of these parameters is varied over wide ranges.

Minimum bearing temperatures resulted (with load, DN value, oil flow, and oil-inlet temperature constant) when a low viscosity, low viscosity-index oil was used. Minimum power rejection to the oil (at a constant bearing temperature) resulted when a low-viscosity oil was introduced at high inlet temperatures.

In the viscosity range investigated, bearing temperatures increased with increasing oil viscosity at constant DN, load, and oil flow. A higher oil flow of a more viscous oil was, therefore, required to maintain a constant bearing temperature. At a constant bearing temperature, DN, and load, the power rejected to the oil increased with increasing oil viscosity. At constant bearing temperature, DN, and load, and when a specific oil was used for lubrication, the measured power rejected to the oil decreased with increasing oil-inlet temperature within the range investigated.
From authors' summary by H. Neifert, USA

3278. Krouse, R., Ball bearing geometry, *Mach. Design* **23,** 2, 158–162, Feb. 1951.

Paper deals with the problem of ball bearings to provide high axial stiffness. In particular, it is concerned with the geometry of angular contact and double curvature ball bearings which may be used for locating shafts axially. Author derives geometrical relations which permit calculation of axial play. Method of taking into account thermal expansion of inner and outer races and the balls is also outlined. Procedure to be followed in determining final geometry is given and an example is worked out. Method should prove valuable where axial stability is important, though knowledge of operating temperatures is essential.
Joseph B. Bidwell, USA

3279. Niemann, G., and Kraupner, K. W., Plastic behavior of revolving steel rollers with point contact (in German), *VDI-Forschungsheft* 434, 1–15, 1952.

Tester used embodies two hardened steel disks pressed, by means of a nutcracker-lever system, against either side of driven shaft, which is the test piece; the disks roll on the shaft. Plastic

deformation and other kinds of damage to rubbing surfaces were studied as functions of number of rolling cycles, hardness, radii of curvature, test load, and preload. Roller bearing experience is confirmed in that there was invariably some amount of plastic deformation, even at the lightest loads.

H. Blok, Holland

3280. Artobolevskiĭ, I. I., Kostitsin, V. T., and Raevskiĭ, N. P., On a certain state of a shaft rotating in a bearing without lubrication and with play (in Russian), *Trudi Sem. teor. Mash. Mekh.* **5**, 19, 5–21, 1948.

The elementary (plane) problem of the motion of a dry journal bearing is solved, taking into account sliding and rolling friction. The condition for purely rolling motion is derived. This motion was analyzed experimentally by (oscillographically) recording the horizontal and vertical displacements of the shaft axis. In a certain rpm range the axis was found to engage in steady motion on the surface of a cylinder coaxial with the bearing, traveling around the cylinder $r/(R - r)$ times per shaft revolution (R, r being the bearing and shaft radii). During this steady motion, the shaft was purely rolling in the bearing. Outside of this range, the axis had a wobbling motion. The range observed agrees fairly well with the pure rolling range computed.

A. W. Wundheiler, USA

3281. Bowden, F. P., and others, A discussion on friction, *Proc. roy. Soc. Lond.* (A) **212**, 1111, 439–520, May 1952.

Fundamental and significant work is reported on three phases of friction: Friction of metals, friction of nonmetals, and boundary and extreme pressure lubrication. In the first category, results reported indicate that, with most metals, natural oxide layer is sufficient to prevent metallic contact at small loads, and that surface roughness, thickness of oxide film, and relative hardness of oxide and metal substrate affect degree of protection afforded bare metal. Coefficient of friction is found to decrease linearly with hardness. Metallic surfaces sliding together become coated with an amorphous or Beilby layer formed when asperities become molten and smear out. Coefficient of friction between clean surfaces is found to be on the order of 25 to 100. Metallic wear is found to be proportional to load and to distance traversed with load pressures below $^1/_3$ the hardness; with loads above this value, wear rate is greatly accelerated.

In the second category, frictional behavior of several plastics and effect of outgassing on frictional properties of diamond, graphite, and carbon are reported. Friction between single textile fibers is reported to decrease with increasing load.

In the third category, desirable characteristics of boundary lubricants and effect of surface condition are discussed together with an interferometric technique for measuring thickness of surface films. Action of extreme pressure lubricants is explained.

Entire discussion constitutes a significant contribution to the knowledge of friction phenomena. W. J. Anderson, USA

3282. Michell, A. G. M., Lubrication. Its principles and practice, London & Glasgow, Blackie & Son, Ltd., 1950, xxi + 317 pp. 35s.

Volume reflects the thoughts and experiences of one who has spent many productive years in the field of lubrication. Following a brief treatment of definitions and fundamental concepts, author discusses viscosity and methods of measuring it. Considerable material is treated in analytic detail, including such unusual items as localized heat generation in oil film, reversed flow in film of a lightly loaded bearing with large inclination and side flow. The difference between friction forces on mating bearing surfaces is held to be an artifax and author therefore adopts a mean value. An extensive section describes a wide variety of slider bearings and the details of their construction. Brief theoretical treatment of journal bearings is followed by extensive descriptive treatment. Unusual types of bearings are included, and considerable space is devoted to water-lubricated bearings. Rolling bearings, gears, worms, and piston rings are next treated, mainly from qualitative point of view. Last two chapters treat distribution and purification of lubricants.

Probably the most significant departure of text is its emphasis on surface irregularities and their role in promoting changes in bearing performance usually attributed to metallic contact and boundary lubrication. Such subjects as dry friction, wear, and extreme pressure lubrication are not treated.

A number of unusual definitions are adopted; such as "rugosities" for surface projections, "fluent film" for more usual fluid-film lubrication, "charge" for a pressure-applied load on a bearing element, and "coefficient of resistance" for coefficient of friction of journal bearing. Numerical results, where given, are in both metric and English units. Viscosities are always expressed in poise instead of in pound, inch, second units as in most engineering texts.

Book is physically attractive and contains a number of large folding charts for design purposes attached to front and back covers. In author's words, purpose of book is ". . . to assist practice." Book certainly should accomplish this aim and be useful to all concerned with bearing design.

Milton C. Shaw, USA

Marine Engineering Problems

(See also Rev. 3053)

3283. Standard nomenclature and symbols for hull and propeller resistance and propulsion strength and vibration, *Brit. Shipbuild. Res. Assn. Rep.* no. 6, 8 pp., 1949.

3284. Niedermair, J. C., Ship motions, *J. Amer. Soc. nav. Engrs.* **64**, 1, 11–25, Feb. 1952 = *Shipbuilder* **59**, 524, 303–306, Apr. 1952.

Ship motions at sea cover a large and increasingly important part of the naval architect's problems. The basic motions referred to are the rotations of roll, pitch, yaw, and the translations of surge, sway, and heave. Effects of slamming and lurching, also, are referred to in the discussion. In this paper, new experimental findings, in correlation with recent theoretical work, are presented in the study of the reduction of speed in waves, pitch, and surge.

The reduction-of-speed problem requires development; and though resistance in rough water has received treatment, the analogous problem of reduction of speed at constant thrust is hereby put to preliminary test, in a small tank, through the facilities of the carefully engineered Newport News Hydraulic Laboratory, equipped with a wave maker. The waves were based on full-scale 300 to 900-ft lengths. A standard, empirical, full-scale height was taken as 1.1 multiplied by square root of feet length, as customary for strength calculations. The model waves, scaled down, do not conform to this. It is believed that less severe waves, for more average conditions of the sea, should be selected so that the (1.1) factor could be reduced to (0.55) for motion studies. Tests show that the maximum speed reduction occurs at wave lengths about equal to the ship length. (This condition does not appear to coincide with that for maximum pitching amplitude.) The calculated and so-called "natural period of the ship" at synchronism (equal to the period of encounter) occurs just prior to the maximum reduction in speed. Possibly the reduction of

speed, as well as pitching amplitude, is not most critical at synchronism, or else the natural periods are (and should be so calculated) slightly longer than as usually calculated. Reductions in observed speed were great, such as from 30 knots to about 15 or 17 knots, full scale. One model showed a similar, numerical reduction from 20 knots to 7 knots. Another set of tests reports maximum speed-with-dry-decks for each wave length, wherein the value of extra freeboard forward is shown. Increased freeboard naturally permits higher speeds in waves, with a dry deck. Further tests, with added superstructures, are suggested. A complete theoretical analysis would have to include varying orbital velocities of waves, slope, drag, and ship-and-wave interference patterns.

In most of the pitching experiments, existing theories are found to be in substantial agreement with the new experimental data, since maximum pitching amplitudes not necessarily occurring at synchronism may be explainable as a function of the exciting forces of the seas, as well as of the relationship between natural periods of ship and encounter with the waves. Exciting forces generally grow with wave length and speed increase. Maximum pitching occurs at wave lengths well beyond those of synchronism.

The natural pitching period has been calculated with regard to inertia of the entrained water, or the added inertia. Data supplied are for deeply submerged bodies, and, lacking data for free surface, the submerged are used here. The theoretical formula for period of pitching includes terms for the mass moment of inertia of the weights, the added mass moment of inertia of the water, and the longitudinal moment of inertia of the water plane; being proportional to the square root of the sum of the first two, and inversely to the square root of the latter. (A damping factor perhaps should be included.) This formula is converted for present use in terms of gyradius of each quantity. The net increase in natural period, over that calculated by neglecting the added inertia, ranges from 42 to 53% for the vessel models tested. It may be that the true added inertia coefficients are still larger than as used. The pitching amplitudes, relative to wave slope (which relation has been questioned) are plotted against wave length. The curves for all the models are nearly the same. The pitching amplitudes are much more a function of the exciting forces of the sea than they are of the design of the ship; and, of course, depend on wave size and ship speed. The pitching periods appear to be not the ships' natural periods, but forced periods. However, regular waves for models do not correspond well with the more usual irregular waves for ships.

Eastman Smith, USA

3285. Telfer, E. V., Marine propeller and propulsion miscellany, *N. E. Cst. Instn. Engrs. Shipb. Trans.* **68,** part 3, 107–132, Jan. 1952 = *Shipbuilder* **59,** 522, 156–163, Mar. 1952.

Very pertinent remarks and suggestions on propulsion problems, model-experiment data, and ship-trials experience disclose author's well-known endeavor to reach better ship-model correlation. First, original views are developed about experimental curves of propeller thrust and efficiency, about scale effects, and about statistical consideration in the tests made in open water. A new interpretation of the relative rotative efficiency of the propeller behind the hull is suggested to bring propulsion scale effects into better understanding, and a correct expression of this factor is derived by use of the apparent propulsive coefficient.

A new method of analyzing ship-trials data is outlined, using a double speed and power curve, with and against weather. Finally, the results obtained on models with stimulated turbulence are analyzed in order to dispose of the current model-experiment dilemma, precipitated by the development of welded shell construction.

R. Spronck, Belgium

3286. Matejka, S., Nomogram for dimensioning of parts of ship hulls (in Swedish), *Tekn. Tidskr.* **81,** 46, 1085–1086, Dec. 1951.

Author proposes a more frequent use of graphical presentation of data in connection with the construction of ships, and gives some examples thereof.

E. Hogner, Sweden

3287. Föttinger, H., Advances in the theory of flow in engineering and shipbuilding, *David W. Taylor Mod. Basin Transl.* 48, 60 pp., May 1952.

Translation from *Jahrb. der Schiffbautechn. Gesellsch.* **25,** 295–344, 1924.

3288. Pagès, Mechanical study of the shock produced in docking a ship at a pier (in French), *Ann. Ponts Chauss.* **122,** 2, 205–217, Mar./Apr. 1952.

It is known that the contact forces incidental to the docking of a ship at a pier or wharf can be damaging. Author presents an elementary analysis of this important problem in mechanical shock.

The equations of motion of the ship as well as the forces and moments acting between the ship and pier are determined from the standpoint of rigid body and particle mechanics. No consideration is given to the problem of ship and pier as distributed elastic media.

The first case treated is normal collision without rebound. A table of energy of absorption for various tonnages is deduced. The second case treated is tangential collision without rebound. Some consideration is also given to frictional forces between ship and pier. In conclusion, a set of rules is presented for signing fittings on docks to reduce shock and prevent damage.

The studies were made for the port authority of Bordeaux and are given in part in the British journal, *Dock and Harbour Authority.*

W. H. Hoppmann, II, USA

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